FRIEDMAN’S CHARACTERIZATION OF THE NATURAL RATE OF UNEMPLOYMENT

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OCTOBER 2014

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20 October, 2014
Revised: 25\textsuperscript{th} October, 2014

\textsuperscript{*} By this I mean a \textit{mathematical} characterization in the usual analytic sense.

\textsuperscript{\textdagger} I am greatly indebted to Ilker Aslanetepe, Geoff Harcourt and Ragu Ragupathy for valuable help in weeding out the inevitable crop of ‘typos’, as well as other useful suggestions, some of which I have incorporated in this revised version. Perhaps I’ll get a chance to expand the ideas in this version in a more nuanced, expanded, argument, placing the Friedman-Lucas \textit{nexus} in the larger context of the development of \textit{Equilibrium Macroeconomics}.
Abstract

Arguably, of the many pioneering classics authored by Milton Friedman, it is his *Presidential Address* to the American Economic Association, in December, 1967, published as *The Role of Monetary Policy* in the AEA, in March, 1968, that may have had the greatest impact in serious policy circles. In this paper I try to discuss its analytical foundations, largely critically, and find the claims mathematically untenable. The setting – the background – for its emergence, and the way it influenced the future course of macroeconomics is also a part of the narrative, although at a much lesser level of rigour. Friedman’s decisive role in the direction which macroeconomics took, in confused era after the demise of the Neoclassical Synthesis, is outlined, albeit with mixed feelings of intellectual sympathy.

**Keywords:** Natural Rate of Unemployment, Natural Rate of Interest, Equilibrium Level of Unemployment, Walrasian System of General Equilibrium Equations, Wicksell, Keynes, Irving Fisher, Phillips Curve.
§ 1. Introduction

Milton Friedman’s Presidential Address to the *American Economic Association*, delivered on 27 December, 1967 (Friedman, 1968), is one of the enduring classics of Macroeconomics of the second half of the twentieth century. It ranks with Paul Samuelson’s overlapping generations classic, Robert Lucas’ pioneering contribution to the genesis of Newclassical economics (Lucas, 1972), Robert Clower’s brilliant indictment of the Neoclassical Synthesis (Clower, 1965) and Richard Goodwin’s supremely elegant, characteristically concise, invoking of the Lotka-Volterra dynamic mechanisms to model the symbiotic nature of capitalist interaction (Goodwin, 1967) as one of five perennial documents of the dynamic development of macroeconomic theory and policy in that half-century.

Paradoxically, for the first fifty years of the twentieth century, it would more be in terms of the classic books that defined the subject that came to be called Macroeconomics after WW II. In this case, I have always chosen Wicksell (1906), Robertson (1926), Keynes (1936), Myrdal (1939) and Lindahl (1939).¹

Many scholars would identify the core arguments in the classic by Friedman as an indictment, refinement and a generalization of the Phillips Curve, as conventionally interpreted and used in macroeconomic policy contexts; others may see and read it as a precursor to the rational expectations revolution, via the suggestion for an expectation augmented Phillips Curve.

Few would deny that its lasting impact on the subject came about via the introduction of the notion of a *Natural Rate of Unemployment* (*ibid*, p. 8). Friedman’s classic – as well as the other nine I have chosen – seems to have suffered the fate of pioneering contributions to a subject: often quoted, seldom read. This is, perhaps, one reason – but there are many - why one re-

¹ Naturally, there will be many with a different set of two-fives, wondering why there is neither Sraffa (1926) or Solow (1956); why neither Fisher (1911), nor Kydland & Prescott (1982) are in the lists; why not Harrod (1939) or Patinkin (1955; 1965) - and what of Frisch (1933) and Robinson (1956), and a hundred others? All I can say is that I have found it possible to consider the above ten classics as the *closure* – in an algebraic sense – of one hundred years (and a little more) of macroeconomics, in that I have found it possible to tell the story of this weird and wonderful subject, entirely in terms of them, given time.
discovers unpolished gems in them, buried in the subtle caveats and the explicitly unstated assumptions.

In this paper I concentrate on the *Natural Rate of Unemployment*, its claimed *analogy* with Wicksell’s celebrated *Natural Rate of Interest*, its asserted analytic determination and, to some extent, the associated economic interpretation. From the dual standpoint of a Wicksellian and a Keynesian, on the one hand, and from the point of view of computable or constructive mathematics, on the other, I am not convinced that the many claims in Friedman’s classic can be substantiated. I believe some of my skepticism can be demonstrated with the kind of rigour that goes along with any mathematical characterization of an analytical claim. However, the doctrine-historical doubts I cast may appear as a form of aspersion from someone who is opposed to the ideological underpinnings of the policy nihilism implied in Friedman’s powerful assertions.

In any case, the paper is structured as follows. In the next section I attempt to anchor the notion of the *Natural Rate of Unemployment* in the way macroeconomics evolved in decade on either side of 1967 – the Phillips (1958) classic, at one end, and the emergence of the Lucasian revolution from 1972 onwards. In section 3, I claim that the mathematical justification of Friedman’s claims, definitions and generalizations are vacuous. In the final section I indulge in some speculative reinterpretations, from both a Wicksellian and a Keynesian perspective, which enables me to cast doubt on Friedman’s policy conclusions – some extensions of which have become the hallmark of Newclassical policy nihilism and other dogmas.

§ 2. The *Natural Rate of Unemployment* and Macroeconomics

In his remembrances of the role he played, simultaneously with Friedman, in the genesis of what eventually came to be referred to as the *Natural Rate of Unemployment* – bracketed, justly, with both their names – Phelps observed:

“My association with that concept goes back to my 1967 essay on optimal inflation* control* [Phelps, 1967], which gave the idea an algebraic formulation. There I dubbed the concept the `warranted' rate of unemployment because, in the model there, it is that

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2 It remains unclear to me, even after many readings over the past thirty years, that this paper was about ‘optimal inflation’, rather than ‘optimal (aggregate) demand’ in a non-stochastic model of the macroeconomy.
unemployment level which is called for if the public's expectations of the rate of inflation are to be met. Since a characteristic of Roy Harrod's 'warranted rate of growth' was that it might be manipulated if otherwise it would cause harm, I thought I had hit upon a value-free term. But Milton Friedman's *catchy term* for the same idea, though derived from a different model, was the easy winner. Not that I (nor Friedman) was the first to conceive or utilize the idea: Hayek, Mises, Fellner, and Wallich all talked about and wrote about it in earlier decades, and the latter two taught it to me. It runs in the blood of economists between the Danube and the Rhine.\(^3\) Phelps, 1979, p.93; second set of italics added.

This curiosum, coupled to the equally little known fact, at least in the standard literature, that Grunberg and Modigliani (1954), used exactly the same Harrodian term\(^4\) warranted expectations, also for exactly the same reasons as Phelps, to describe what has come to be known as rational expectations, has always signified for me that a disequilibrium between a *warranted* and a *natural* rate must lie at the heart of any decent aggregative dynamic model. A rate of unemployment in the labour market – essentially, in Harrodian terms, involuntary unemployment with all that such a notion implies for a Keynesian interpretation of the Macrostatics - is warranted by the expectations of profitability on the basis of which optimizing behavior on

\(^3\) The penultimate phrase and the last sentence are, surely, entirely gratuitous. Whether Hayek or Mises (in the inclusive sense), were economists whose ‘blood’ was genetically endowed with the notion of the natural rate of unemployment – or in its incarnation as a warranted rate in the sense of Harrod – because it emerged from them being ‘between the Danube and the Rhine’ is a debatable question. In the case of the former, it would depend on the side of the city of Vienna that was his home; in the case of the latter, it is even more tenuous, given that Mises was born in (today’s) Ukraine! As for Fellner and Wallich, the assertion is simply false – Fellner was born in Budapest and studied at Berlin and Zurich, before emigrating to the US; Wallich was born and educated in Berlin and arrived in the US by a circuitous route that did not involve any sojourn ‘between the Danube and the Rhine.’ Of course, there may have been many economists in whose ‘blood’, formed ‘between the Danube and the Rhine’, ran the notion of the ‘warranted rate’ or the ‘natural rate’ or whatever ‘rate’; but for every such economist, it is not unlikely that an equally – or more distinguished – scholar in whose ‘blood’, formed ‘between the Danube and the Rhine’, no such notion was intrinsic.

\(^4\) As Grunberg & Modigliani (ibid, p. 469; italics added), remarked:

"There exists, then, at least one correct public prediction, provided that the supply and demand curves intersect once in the positive quadrant. Note that in our example public prediction prevents possible error of expectation on the part of suppliers. As suppliers fully accept the public prediction - which turns out to be correct - they act on the basis of *warranted expectation.*"

This paper used the Brouwer fixed-point theorem, inspired by Herbert Simon (ibid, footnote 1, p. 465), slightly prior to, and independent of, its use by Arrow and Debreu (1954).
investments and choice of techniques will imply a particular level of employment and, via the effect on factor shares, effective demand such as to lead to dynamics in the product market. This, in turn, will feedback on the dynamics of the labour market and, by means of a warranted rate of expectations of profitability, lead to a next round of impact on investment and choice of techniques as a result of productivity changes and the cycle continues, either towards a stable disequilibrium dynamics in the form of a point attractor or a fluctuating profile in any one of many possible basins of attraction of limit configurations: limit cycles, strange attractors, etc.

The key unorthodox adjustment dynamics in this scenario is provided by freeing factor shares to vary over the cycle. Any use of a conventional production function locks the exponents in such a way that factor shares are prevented from acting as adjustment variables in mediating between the imbalances in aggregate supply and demand in the labour and product markets. However, by using the more general and more flexible technical progress function it is possible to endow the distributive variables with a more active role in the overall disequilibrium dynamics of a model of fluctuations in product and labour markets.

All of this is comprehensively ignored in the Friedman classic which, helped codify the emergence of Newclassical economics via Lucas (op.cit). Lucas, most deftly, put together seven elements which characterize the closure of Newclassical economics: Rational Expectations (RE), the Natural Rate of Unemployment (NRU), Search in the Labour Market, Neoclassical Growth Theory, the Phelpsian Island Paradigm⁵, the incorporation of Human Capital to augment the traditional two factors in an aggregate neoclassical production function, and underpinned by theoretical policy anchors in the two fundamental theorems of welfare economics, all of this held together in an equilibrium framework of optimum decision-making of some sort. The role of money, emphasized – in a way – in the original Lucasian formulation, more or less disappeared in the fully fledged DSGE model of more recent vintage, where neutrality, in one form or another, resurrects the Hayek-Friedman vision, almost with a vengeance.

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⁵ Essentially to drive a necessary wedge between local and global information, in a system where agents are signal processors, inhabiting Phelpsian Islands, filtering out noise and basing optimal decisions on the information that is sanctioned by Kalman’s ingenious device. These signal processors are a trivial subset of the computably underpinned Information Processing Systems of Human Problem Solving by Newell and Simon (1972).
Whether Friedman himself approves of this particular ahistorical, nihilistic policy framework, remains a moot point. I myself do not think modern macroeconomics, however much it owes some of its building blocks to Friedman’s pioneering suggestions – in addition to embracing his instrumentalist methodology of ‘as if’, often implicitly – is something with which Friedman would have been overly comfortable. He was a neoclassical in the Marshallian mould, however much an anti-Keynesian he professed to be.\textsuperscript{6}

§ 3. Formal Characterization of the \textit{Natural Rate of Unemployment}

In Arthur Seldon’s wholly enthusiastic endorsement of Friedman’s IEA lecture and monograph (Friedman, 1975), we are informed that the ‘\textit{locus classicus}’ that is the Friedmanian definition of the term \textit{Natural Rate of Unemployment},

\begin{quote}
\text{“[S]hows .. that analysis must precede decision, theory must inform practice, and, as always, ideas will determine action.”}
Seldon, in Friedman (\textit{ibid}), p. 8.
\end{quote}

But what if the \textit{analysis is seriously fallacious} – from claimed formal assertions and the theory is fundamentally a disguised form of the Marxian theory of the ‘reserve army of labour’ and the ideas that determine action echo that famous Keynesian dictum (Keynes, 1936, pp. 383–4; italics added):

\begin{quote}
\text{“[T]he ideas of economists and political philosophers, both when they are right and when they are wrong, are more powerful than is commonly understood. Indeed the world is ruled by little else. Practical men, who believe themselves to be quite exempt from any intellectual influences, are usually the slaves of some defunct economist. Madmen in authority, who hear voices in the air, are distilling their frenzy from some academic scribbler of a few years back. I am sure that the power of vested interests is vastly exaggerated compared with the gradual encroachment of ideas. Not, indeed, immediately, but after a certain interval; for in the field of economic and political philosophy there are not many who are influenced by new theories after they are twenty-five or thirty years of age, so that the ideas which civil servants and politicians and even agitators apply to current events are not likely to be the newest. But, soon or late, it is ideas, not vested interests, which are dangerous for good or evil.”}
\end{quote}

\textsuperscript{6} Yet, it was Friedman who was the originator of the phrase ‘we are all Keynesians now’, although the full sentence in which this was a phrase is: \textit{In one sense, we are all Keynesians now; in another, nobody is any longer a Keynesian} Friedman, Letter to \textit{Time}, February, 4, 1966.
The main point of this section is to show, rigorously, that the analysis – and the associated claims for policy – in the Friedman classic is not only fallacious but algorithmically vacuous. This latter issue makes the framework inapplicable in any policy context.

I consider the two propositions embedded in the following two celebrated, oft quoted (seldom, in my opinion, investigated in any rigorous way whatsoever), assertions by Friedman to be fallacious and cannot be substantiated by any formalization (Friedman, 1968, p. 8 & footnote 3, p. 8, respectively; italics added):

i. “The ‘natural rate of unemployment,’ …, is the level that would be ground out by the Walrasian system of general equilibrium equations”, provided there is imbedded in them the actual structural characteristics of the labor and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labor availabilities, the costs of mobility, and so on.”

ii. It is perhaps worth noting that this ‘natural’ rate need not correspond to equality between the number unemployed and the number of job vacancies. For any given structure of the labor market, there will be some equilibrium relation between these two magnitudes, but there is no reason why it should be one of equality.”

It is clearly evident that the italicised assertion in (ii) is a formal existence theorem – or, at least, a ‘proposition’ – while (1) is an unambiguous claim that the Walrasian system of general equilibrium equations can grind out a solution – whether with or without the additional features Friedman considers desirable (‘the actual characteristics of the labour and commodity markets’ – whatever ‘actual’ means, in this or any other case – ‘ditto’ for all of the other ‘caveats’).

Now, what does ‘ground out’ mean – formally or informally? Let us assume that it means ‘solvable’, in the usual, ‘common sense’, notion of the term, whether by ‘brute force’ or more refined methods (which will, of course, depend on the nature and number of these ‘equations’). Is the Walrasian system of general equilibrium equations, in fact, solvable? It is easy to show, as rigourously as one wishes – in any sense of rigour one wishes to use - that it is not, provided

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7 Friedman does not refer to a ‘system of inequalities’.
8 The notions of solvability and unsolvability require formal definitions to make sense in this – or any other – serious context. To the best of this author’s knowledge, the most detailed consideration of the notion of solvability, especially in an economic decision-making context, is that by Newell & Simon (op.cit).
9 As G.H. Hardy wryly noted (Hardy, 1929, p. 18; italics in the original):
we understand the difference between proving the existence of a solution and finding a method to demonstrate the solution. This is nowhere better clarified than in the prescient observation by Arrow, et. al., (1958, p. 17; italics added):

“[W]e want to stress that solutions which are not effectively computable\(^\text{10}\) are not properly solutions at all. Existence theorems and equations which must be satisfied by optimal solutions are useful tools toward arriving at effective solutions, but the two must not be confused.”\(^\text{11}\)

In this precise sense of solvability, I have shown in a series of articles\(^\text{12}\) that no formalisation of the Walrasian system of equilibrium equations is either constructively or computable solvable; a fortiori, then, this conclusion holds true of Friedman’s entirely justifiable augmented system of such equations — or inequalities — incorporating realistic (sic!) market imperfections, stochastic variability ‘and so on’.

As for the claim in (ii), i.e., ‘for any given structure of the labor market, there will be some equilibrium relation between the number unemployed and the number of job vacancies,’ interpreted as a pure existence theorem is easily demonstrated in a classical\(^\text{13}\) model of a growth cycle, as in Goodwin (1967). However, quite apart from the well-known mathematical

"[T]here is, strictly, no such thing as mathematical proof; that we can, in the last analysis, do nothing but point; that proofs are what ... I call gas, rhetorical flourishes designed to affect psychology, pictures on the board in the lecture, devises to stimulate the imagination of pupils. This is plainly not the whole truth, but there is a good deal in it.”

I am, unfortunately, not able to interpret the above two assertions by Friedman as anything other than ‘devices to stimulate the imagination’. There is not the slightest hope, without going back to the proverbial ‘drawing board’ of Walrasian general equilibrium theory, to redo it ‘nearer one’s heart’s desire’ (pace Khayyaam). One may have expected a Marshallian like Friedman to appeal to this Cambridge neoclassical maestro, than to his almost contemporary Lausannian, Walras – especially in this aspect.\(^\text{10}\) The careful reader must interpret effective in the sense of computability (or, recursion) theory.\(^\text{11}\) It is pity, therefore, that the classic text by Arrow and Hahn (1971) is replete with such ‘confusions’.\(^\text{12}\) See Velupillai (2006, 2009, 2013 & 2014), I have shown, with customary mathematical rigour (sic!) that the standard framework of General Equilibrium Theory is neither computable nor constructive.\(^\text{13}\) In the sense of Marx, not Keynes, and hence the ‘equilibrium relation’, interpreted by Goodwin (op.cit, p. 58) in the sense of a Marxian ‘reserve army of labour’. It is equally easy to demonstrate the existence of such an ‘equilibrium’ relation involving involuntary unemployment in a strict Keynesian sense.
infelicities of this model, it is also squarely within the tradition of assuming a variant of Say’s Law – as in the Friedman classic\textsuperscript{14}. Here, too, it is easy to show – with all the necessary rigour one demands – that removing these two infelicities entails the loss of any simple ‘equilibrium relation’ between the number unemployed and the number of job vacancies.’

\textbf{§ 3. Reflections and Concluding Notes}

Although the \textit{Phillips Curve}, in its several incarnations at the time Friedman delivered his celebrated lecture, loomed large as a backdrop against which he developed his notions of the \textit{Natural Rate of Unemployment} and its existence as an equilibrium, \textit{long-run}, relation between ‘the number employed and the number of job vacancies’, I have eschewed any appeal or reference to it\textsuperscript{15}. I have refrained from doing so for one specific reason: my aim was to show that even on its own, these notions and claims are quantitatively meaningless. In particular, there is no justification whatsoever to ground any kind of implementable monetary policy with these notions underpinning the model that is used for this purpose – with or without rational expectations, with or without incorporating more ‘realistic’ structural characteristics of the labour market, and so on.

The Lucasian revolution has, however, all but decimated the visions and prescriptions, for good and evil, propagated by Keynes and Friedman.

The benchmark model of the macroeconomy has no role for money, let alone monetary policy. All that ‘constrained discretion’ is supposed to achieve, even when coherent and applicable, is a rational path towards the \textit{real} dynamic stochastic general equilibrium (DSGE), prescribed by the currently dominant theory of macrodynamics, whether of the Newclassical or of the New Keynesian variety.

\textsuperscript{14} Some of the disequilibrium dynamics statements I made, above, in the previous section, must be read with due caution for this aspect.

\textsuperscript{15} This may also imply a sanctification of one or another form of \textit{Say’s Law} (see Pasinetti, 1997, especially pp.101-2), the implication being the distinction between the \textit{long-run} and the \textit{short-run} is obliterated.
Many of us continue to rest our analytical visions, for macroeconomic theory and monetary policy, on the foundations built by Keynes and Friedman (themselves relying on Wicksellian precepts) in the hope that the Robertsonian ‘hunted hare’\textsuperscript{16} will eventually return to \textit{its origins}, even after any number of peripatetic wanderings.

The origins are invariably in the Wicksellian classics. It may well be churlish to end on a note of serious skepticism, regarding Friedman’s flippant – this word is chosen with deliberate intention-references to, and use of Wicksellian terms, words and theories. I am not able to infer any deep anchoring in the Wicksell classics by Friedman and, therefore, his appeal to, and use of a variety of that great Swede’s innovative ideas is less than felicitous. In particular, it is my considered opinion that Friedman is seriously misguided in ‘borrowing’ Wicksell’s tortured concept of the Natural Rate of Interest, without the slightest indication that the notion, at the hands not only of the originator of the notion, but also by a host of distinguished Swedish contemporaries – David Davidson, in particular – and his ‘second generation’ followers, especially Lindahl and Myrdal, was refined in many and usefully measurable ways. This is especially the case with Myrdal (1939), as is confirmed by the remarkable revised-review by Hicks (1934, 1982)\textsuperscript{17}.

What I find even more distressing is that even Friedman’s addition (\textit{op. cit}, p. 8), ‘of only one wrinkle to Wicksell – the Irving Fisher distinction between the nominal and the real interest rate of interest’,\textsuperscript{18} would not have been necessary had the great monetarist studied the Swedish literature – even if only those available in the English language – more diligently.

All the same, I would personally have much preferred to have Milton Friedman’s outspoken and always innovative visions and pungent opinions – whilst holding his monetary, quantity theoretic,

\begin{itemize}
\item \textsuperscript{16} In inimitably enchanting Robertsonian prose (Robertson, 1954, p. 189): “Now, as I have often pointed out to my students, some of whom have been brought up in sporting circles, highbrow opinion is like a hunted hare; if you stand in the same place, or nearly the same place, it can be relied upon to come round to you in a circle.”
\item \textsuperscript{17} In his reflections on the original review, Hicks wrote (\textit{op. cit}, p. 43): “[T]hough I praised the book highly, I have come to feel that I did not praise it highly enough.”
\item \textsuperscript{18} This is quite apart from my serious doubts that Friedman has even understood the analytical subtleties in Fisher – not all quite so mathematically or economically sensible or consistent – as pointed out brilliantly by Adarkar’s little known gem (Adarkar, 1934).
\end{itemize}
grounds, in the particular sense of the above Robertsonian ‘hunted hare’ – on monetary policy, ‘warts and all’, as the guardian angel of modern macroeconomics rather than the antiseptic alternatives we are confronted with, these days.
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