A Slip - or Two - in MIP

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· *MIP*, of course, refers to *Money, Interest and Prices* by Patinkin (1965). I shall, in the sequel, refer always to the second edition of this classic, although the issue discussed in this brief paper is relevant for *all* of the editions of *MIP* – the first (1956), second and also the ‘abridged’ version of the second edition (1989). The latter edition (abridged ‘only’ to the extent that the last, *Supplementary Notes and Studies in the Literature*) included, ‘reproduced unchanged’, the *Mathematical Appendix*. The main Slip in *MIP* appears in this *Appendix*, although there are other (mathematical) slips, which are discussed less comprehensively. I am greatly indebted to my critical friends, Dr. Patrick Love, of the OECD, and Dr. Francesco Luna, of the IMF, for comments and encouragement of an earlier draft of the paper. Naturally, the usual caveat applies.
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Abstract

In this paper an attempt is made to correct a slip – or two – in Patinkin’s perennial classic, *Money, Interest and Prices* (*MIP*). The aim is constructive, in the sense that such a ‘correction’ keeps the book ‘alive’ in its masterly pedagogical role, in which it has enlightened several generations of monetary macroeconomists – if not others, too. The suggested correction also re-validates that other valuable conceptual tool, the *Correspondence Principle*. However, some doubt is cast upon the thought experiment that underpins the ‘construction’ of an excess demand function and, on its basis, the sequential approximation of a monetary – or money – to a barter economy.

**JEL Codes**: B 31, C 62, D 50, E 43, E 49

**Key Words**: Correspondence Principle, Excess Demand function, *Mechanical Tatônnement*, Dynamical Systems, Multiple Equilibria, Heteroclinic orbit
§ 1. A Preamble

Approximately one hundred years spanned the appearance of Wicksell’s two classics (Wicksell, 1898, 1906) on Monetary Macroeconomics and that of Woodford’s ‘Neowicksellian’ (cf. Hoover, 2006) tome (Woodford, 2003). It will, surely, not be an exaggeration to state that Fisher (1907, 1911), Keynes (1930, 1936) and Friedman (1956, 1963) were the only pioneering works that had more influence in the policy and pedagogical practice than Patinkin’s (still) influential Money, Interest and Prices (Patinkin, 1956, 1965, henceforth referred to as MIP), in a remarkable century of developments in monetary theory and the theory of monetary policy. Indeed, a strong case can be made that a whole generation – or even two – of graduate students were introduced to the classics by Wicksell, Fisher and Keynes through having to grapple with the intricate arguments in MIP².

However, in the end, I believe the perceptive skepticism of Lucas (2004, p. 15; italics in the original) is justified:

“But [MIP] doesn’t quite come off, does it? … What are the predictions of Patinkin’s model? The model is too complicated to work them out. All the dynamics are the mechanical auctioneer dynamics⁵ that Samuelson introduced [in Samuelson, 1947], where anything can happen.”

In this paper other reasons why ‘MIP doesn’t quite come off’ are discussed, via two explicit examples. The first, in the next section, is the fundamental reliance of MIP on the theory of ‘classical’ dynamical systems theory⁵ for the phase plane analysis of local dynamics, which – though underpinned by sound fundamentals of market dynamics – is inappropriate for the global monetary policy conclusions drawn’ (MIP, Ch. 12 & Mathematical Appendix c, to Chapter 12). This reliance results in a (mathematical) slip, for which an economically motivated correction is suggested. Secondly, allied to the first, is an incorrect indictment (MIP, p. 499) of the correspondence principle. This is an additional topic in § 2. The brief concluding section

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1 This is, of course, a reference to Friedman & Schwartz (1963).
2 This is not to say that I subscribe to the view that MIP portrayed faithfully the ideas and frameworks of Wicksell, Fisher or Keynes; but it was in trying to understand the claims in MIP that I was myself forced to try to master this triptych of classics, at least partly.
3 ‘Mechanical Tatônnement’ may have been a more accurate phrase.
4 There is a third, but mentioned only en passe, in a footnote in the concluding section.
5 Essentially, as correctly perceived by Lucas (op.cit), on the mathematics of the Foundations of Economic Analysis (Samuelson, 1947, henceforth referred to as FoEA).
contains an outline of the second example, that on MIP as a repository of the perennial neoclassical fulcrum of *neutrality*, as well as tentative thoughts on the relevance of *MIP* for today’s pedagogy, if not also a framework for current monetary macroeconomics.

§ 2. Two Slips in *MIP*

The main slip in *MIP* refers to the ‘perverse’ case – to be described in strictly economic terms below – of the normal market dynamics described in Figure XII-6 (p. 286, *MIP*), depicted in Figure A-8 (p. 502, *MIP*). For convenience the latter figure is reproduced as Figure 1, below (I have added the symbol \( Q \) to denote the ‘equilibrium’ point – which is the ‘villain of the piece’):
Given the assumptions made in \textit{MIP}^6, the (local) dynamic analysis in the $p$-$r$ space can be ‘restricted to the bond and commodity markets’, whose ‘equilibrium conditions are, respectively’:

\begin{align*}
B \left[ Y_0, \frac{1}{r}, \frac{M}{p}, h(p) \right] &= 0 \quad \ldots \ldots \ (1) \\
F \left[ Y_0, r, \frac{M}{p}, h(p) \right] - Y_0 &= 0 \quad \ldots \ldots \ (2)
\end{align*}

where, $h(p)$ : an index of the distribution effect which results from a price change; and: $h'(p) > 0, B_4 > 0, F_4 < 0$;

To highlight the economic mechanism and market dynamics in the $p$-$r$ space, a simplified and a less ‘cluttered’ version of Figure 1 would be as shown in Figure 2, \textit{but with the (local) vector fields in the ‘normal’ senses, in regions ii & iv} \textsuperscript{7}(compared to those in Figure 1):

Denoting by:

\textit{ES} \textsuperscript{i} : \textit{Excess Supply} in Market \textit{i} & \textit{ED} \textsuperscript{i} : \textit{Excess Demand} in Market \textit{i};

\textsuperscript{6} The variables $Y, r, M,$ and $p$ denote, as usual, the aggregate gross real national product, the rate of interest (with its reciprocal denoting the unit price of a bond), money holdings and the general price level of commodities, respectively.

\textsuperscript{7} The attentive reader of \textit{MIP} would have noticed that $Q$ (in Figure 1) is NOT a saddle point equilibrium; and this is so also for Figure 2.
\( i = B, C \), where: \( B \) : Bond Market; \( C \) : Commodity Market.

Now, all the elements to discuss the main slip in \( MIP \) are in place. First of all, referring to Figure 1, the (local) dynamics, as implied by the assigned vector fields in the neighbourhood of the (local) equilibrium, \( Q \), signifies that it is both – simultaneously - **stable and unstable**; the former with respect to regions ii & iv (in the notation of Figure 2) and the latter in the remaining two regions. This is impossible in classical dynamical systems (of the kind lying at the basis of the mathematics of \( \text{FoEA} \)).

Secondly, in Figure 2, the equilibrium is **uniformly** (locally) **unstable**, as can easily be verified in terms of standard market dynamics imputed to these regions. The key reason for the slip in \( MIP \) is due to the apparent paradox of excess supply in the commodity market (\( \text{ES}^C \)) driving interest rates \textit{down}, and excess demand in the bond market (\( \text{ED}^B \)) driving prices \textit{up}. This asymmetry is disallowed, only by the constructed geometry in \( MIP \), in Figure 1.

Thirdly, the ‘coexistence’ of the two kinds of equilibria – stable and unstable, in the neighbourhood of \( Q \) – needs the **theoretical technology** (Lucas, 1982, p. 10) of **heteroclinic** orbit dynamics, linking them. However, in purely economic terms, without resorting to the esoteric mathematics of modern dynamical systems theory⁸ it is possible to proceed as shown in Figure 3.

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⁸ Although these **theoretical technologies** were available at the time \( \text{FoEA} \) or \( MIP \) were written, because Poincaré (1899), had considered these issues more than a century ago, they did not become even part of modern dynamical systems theory till Smale (1967) codified the field using modern terminology.
Referring, firstly, back to Figure 2, in view of the negative slope of B’B’, it is clear that the real balance effect in the bond market, due to a price rise, for example, is dominated by the distribution effect brought about by the assumption, say, of differential savings propensities – leading, thus, to asymmetric behavior in the commodity and bond markets. Thus is motivated the instability of the (local) equilibrium.

The economic essence of instability is, of course, when the ‘law of supply and demand’ works perversely: driving prices up when there is excess supply and driving prices down, when there is an excess demand. In the present context, this ‘perversity’ is generated by switching around the normal effects of the real balance process and the distributional mechanism.

But, of course, such an anomaly or, perhaps, an abnormality, comes to an end by a breakdown in economic activity; or, more likely, due to an ‘endogenous’ mechanism that restores the normal ranking between real balance and distributional effects. This, I believe, would have been Patinkin’s own preferred way out, as he did with the instability of Wicksell’s cumulative process.

Such a way out implies multiple equilibria and nonlinearities and, diagrammatically, is easily illustrated by generating phase-plane dynamics in the p-r space where an unstable equilibrium, \( Q \), is flanked by two stable equilibria, \( R & S \).

Finally, Patinkin’s indictment of the validity of the correspondence principle\(^{10}\) (MIP, p. 499), with respect to \( Q \) in Figure 1, is invalid:

“[D]ynamic analysis does not provide the necessary additional information about comparative statics analysis: the ‘correspondence principle’ does not work.”

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\(^{9}\) The ‘path’ joining the two qualitatively different equilibria would, in a more comprehensive global dynamical systems theoretical perspective, would be considered a heteroclinic orbit.

\(^{10}\) This venerable principle was first, albeit implicitly, enunciated – in economics – by Hicks (1939, p. 62). Later, it was given the name it now has by Samuelson (1947). The origin of the phrase itself probably goes back to Niels Bohr, when that legendary Physicist tried to find a way to reconcile classical and quantum mechanics, in the same way we, in economics, attempted, sometimes unsuccessfully, to obtain fruitful dynamic results from (comparative) static hypotheses cf. Velupillai, 1973). Patinkin’s MIP is essentially an application – and a fruitful one, at that, of the comparative static method. There is, as Lucas correctly intuited (Lucas, op. cit), very little meaningful dynamics in it.
But this is entirely due to the fact that Patinkin assumes the validity of the ‘correspondence principle’ for ‘ambiguous’ equilibria – the best way I can think of describing the status of $Q$, in Figure 1. The domain of application of the classic Hicks-Samuelson ‘Correspondence Principle’ presupposes an unambiguous equilibrium, to which stability conditions are imputable. This is not the case with $Q$.

§ 3. Brief Concluding Notes

Despite – or, perhaps, because – of the infelicities and ‘slips’ in $MIP^{11}$, it remains a classic, even if wounded and partly outdated by developments in theoretical technologies and conceptual advances in the subject of monetary economics, particularly in its macroeconomic variants. It remains a pedagogical classic – all 700 pages of the unabridged second edition – at least as a valuable record of the path taken by monetary macroeconomic theory and policy and, above all, a priceless (sic!) sourcebook, introducing the beginning graduate students – and I dare say also the modern, young, teachers – to the classics of Wicksell, Keynes, Hicks and Samuelson.

It is no wonder, then, that $MIP$ is considered one of the fountainheads of the microfoundations of Keynesian macroeconomics$^{12}$, where involuntary unemployment is a fulcrum around which disequilibrium dynamics is generated.

The perennial fault of any disequilibrium dynamic approach is its intrinsic inability to avoid Mechanical Tatònements. It is in this that the main weakness of $MIP$ lies. In the opinion of this writer, it cries out for an updated, by modern theoretical technologies, edition. But the virtue of all classics is to leave room for the followers, a generation, or two, later, to undertake that arduous task. It is possible, although I am not sure, that this was what Interest and Prices (Woodford, 2003) achieved. After all, the New Keynesians are a ‘modern’ equivalent of the New Classical Synthesis – whether, occasionally, referred to as Neowicksellians, or not.

$^{11}$ The fertile ‘thought experiment’ of imagining a utility computer to generate the excess demand function is another casualty of progress in theoretical technologies (cf, Velupillai, 2013, 2014).

$^{12}$ Especially in its Neoclassical Synthesis variants, although the first edition of $MIP$ was prepared before the third edition of Samuelson’s popular introductory Economics textbook, where the now infamous phrase was first coined and made known to the wider economic community.
But my own interpretation of the way MIP has survived a half-century of vicissitudes in monetary macroeconomics, is through its adherence to the concept of neutrality – more specifically, neutral money. MIP is in that perennial neoclassical tradition that, in its modern age, spans the seven decades between Hayek (1931) and Lucas (1996), where neutrality – neutral money – is the binding force that links a barter and a money economy. Nowhere is MIP more explicit on this issue than when one reads in MIP (p. 75; italics added):

Strictly speaking, … neutrality obtains if the mere conversion of a barter economy to a money economy does not affect equilibrium relative prices and interest. [B]ecause the systems of excess-demand equations of these two economies differ so fundamentally … it is difficult, if not impossible, to make such a comparison in a general way. If, however, we conceive of a barter economy as the limiting position of a money economy whose nominal quantity of money is made smaller and smaller, we can obviate this difficulty. For we can then remain within the system of excess-demand equations of a money economy and note what happens to its equilibrium values of relative prices and interest as the quantity of money approaches zero as a limit. … [W]e can in this way get as ‘close’ as we want to a barter economy without affecting the equilibrium values of these variables.

This interpretation … has one serious drawback: As the nominal quantity of money approaches zero, so does the price level – and at the same rate. Hence the real quantity of money remains unaffected. Thus the limiting position that we have defined as a barter economy is one in which there exists the same real quantity of money as in a money economy! This drawback notwithstanding, there does not seem to any other meaningful way of comparing the respective equilibrium positions of a barter and a money economy.

In this sense it shares the monetary policy nihilism that unites the visions of a Hayek or a Lucas, and is antithetical to the enlightened monetary policy activism of Wicksell and Keynes.

MIP’s theoretical failures are due to the impossibility of the thought experiment envisaged to construct excess demand functions and, thus also the chimera of money as a veil, waiting to be unveiled, so that the real (sic!) economy can be analysed without the encumbrances that are associated with a Wicksellian or Keynesian monetary economy, eschewing Say’s law and extolling the pervasive nature of the fallacy of composition.

But this ‘failure’ is also the reason why it remains a perennial classic, for good – and bad.

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13 At this point Patinkin refers to Hayek (1931), pp. 129-130.
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