Algorithmic Social Sciences Research Unit

ASSRU

Department of Economics
University of Trento
Via Inama 5
381 22 Trento Italy

DISCUSSION PAPER SERIES

2 – 2011/II

THREE CONTRIBUTIONS TO: *ALAN TURING – HIS WORK AND IMPACT* ♠

K. Vela Velupillai♣

JUNE 2011

> "Quantum mechanists always seem to require infinitely many dimensions; I don't think I can cope with so many – I'm going to have about 100 or so – that ought to be enough don't you think?"
>
> Alan Turing to Robin Gandy♦

♠ Forthcoming in: **Alan Turing – His Work and Impact**, edited by S. Barry Cooper & Jan Van Leeuwen, Elsevier, 2011.

♣ I am greatly indebted to Barry Cooper ,not only for inviting me to contribute to this wonderfully appropriate intellectual initiative to celebrate one of the great innovative thinkers of the 20th Century, but also for inspiring me with his own elegant writings on computability theory and mathematical logic. In my three contributions I have not included, in the respective reference lists, any of Turing's own contributions to which I refer. This is simply because I expect all the relevant Turing papers will be properly included in a general list in the book

♦ **Collected Works of A.M. Turing – Mathematical Logic**, edited by R. O. Gandy & C.E.M. Yates North-Holland, Amsterdam, p. 266.

I. Reflections on Wittgenstein's Debates with Turing during his *Lectures on the Foundations of Mathematics*[3]

Andrew Hodges (2008), recalled Max Newman's characterisation of Alan Turing as 'at heart more an applied than a pure mathematician', and went on (p.4; italics added):

> "It might be more true to say that Turing had resisted this Cambridge classification from the outset. He attacked every kind of problem – from *arguing with Wittgenstein*, to the characteristics of electronic components, to the petals of a daisy."

This prompts me to return to Turing's 'debates' with Wittgenstein – now remembering Max Newman's characterisation - during the latter's *Lectures on the Foundations of Mathematics* (Wittgenstein, 1939 [1976]). It is little realised – indeed, to the best of this writer's knowledge, never mentioned – that when Turing attended these lectures, in the Lent and Easter terms of 1939, he was the young (Turing was not quite 27 years old and Wittgenstein turning a *vintage* 40!) author of *Systems of Logic Based on Ordinals* (**Proceedings of the London Mathematical Society**, Series 2, Vol. 45, 1939) where 'ways in which systems of logic may be associated with constructive ordinals' (couched in the language of the λ-calculus) was a main theme. Feferman, in his perceptive *Preface* to '*Systems of Logic*' (Turing, 2001, p.79) observed, correctly in my opinion: Turing never tried to develop an over-all philosophy of mathematics …'. Yet, he (Turing) was engaging one of the great philosophers of the 20th Century on *his* (Wittgenstein's) interpretation – even 'deconstruction' – of Cantor's work on *Transfinite Numbers*!

It is a pity that in these famous lectures by Wittgenstein, Turing was 'set up' as the 'strawman' representing orthodox mathematics and mathematical logic, defending the conventional notion of consistency (not related to its specialised version in the Gödel-Rosser work) in mathematics. Had the protagonists been privy to the Newman-Hodges picture of

[3] I write as an economist who is painfully aware that uninformed references to this dialogue between one of the great philosophers and the pioneer of computability recur in the methodological literature of economics (cf, for example the muddled invoking of a particular part of this famous dialogue by McCloskey (1991), pp. 13-4).

Alan Turing, who ‘began (and ended) with the *physical world*’ (Hodges, op.cit., p.4), the subsequent misrepresentation of Wittgenstein’s stance[4] may have been prevented.

The context for the particularly (in)famous part of the Wittgenstein-Turing dialogue on consistency/contradiction in mathematics (and mathematical logic), it may well be useful to quote the original few remarks in Wittgenstein (op.cit, pp.211-2, Lecture XXII; italics added):

> It was suggested last time [i.e., Lecture XXI] that the danger with a contradiction in logic or mathematics is in the application. Turing suggested that a bridge might collapse.
>
> Now it does not sound quite right to say *that a bridge might fall down because of a contradiction* [in logic or mathematics].”

Now, to place this in proper historical perspective, compare Stanislaw Ulam’s dialogue with Gian-Carlo Rota on collapsing bridges and logical contradictions (Rota, 1986, p.2; italics added):

> “However, out of curiosity I [Rota] decided to play devil’s advocate, and watch his reaction.
> But if what you [Ulam] say is right, what becomes of objectivity, an idea that is so definitively formalized by *mathematical logic* and by the theory of sets, on which you [Ulam] yourself have worked for many years of your youth?
>
> There was a visible emotion in his [Ulam’s] answer. Really? What makes you [Rota] so sure that *mathematical logic corresponds to the way we think*?[5] You are suffering from what the French call a ‘deformation professionelle.’ *Look at the bridge over there. It was built following logical principles. Suppose that a contradiction were to be found in a set theory. Do you honestly believe that the bridge might them fall down?*
>
> Do you [Ulam] then propose that we give up mathematical logic? said I [Rota], in fake amazement.

---

[4] Most egregiously represented by Charles Chihara (1977), only partially blunted by Shanker’s brilliant counterattack (1987, especially Chapter 6, §3).

[5] Brouwer had been there, and Wittgenstein may have remembered it, long before them, and had remarked, in his Inaugural Lecture of 1912 (Brouwer, 1913, p. 84; italics added), most perceptively:

> “To the philosopher or to the anthropologist, but *not to the mathematician*, belongs the task of investigating why certain systems of symbolic logic rather than others may be effectively projected upon nature. *Not to the mathematician*, but to the psychologist, belongs the task of explaining why we believe in certain systems of symbolic logic and not in others, in particular *why we are averse to the so-called contradictory systems* in which the negative as well as the positive of certain propositions are valid.”

> Quite the opposite [said Ulam]. *Logic formalizes only very few of the processes by which we actually think*[6]. The time has come to enrich formal logic by adding to itsome other fundamental notions[7]."

In the years before Laurent Schwartz elegantly encapsulated the *Dirac delta function*[8] with his notion of generalized functions, von Neumann had 'banished' it from 'official' use in physics and quantum mechanics for being mathematically 'improper'. Meanwhile, physicists, with princely unconcern for the prestigious embargo placed on the delta function, went on happily using it for calculations. Engineers, of course, were blissfully unaware of von Neumann's prestige or embargo and went on calculating with the Heaviside operational calculus.

So far as I know, neither the Feynman Integral, nor Bishop's constructivism, have been axiomatised. This has not prevented perfectly valid calculations using Feynman integrals in quantum electrodynamics. For all we know, there are, lurking in the inner recesses of the Platonic Universe, the eventually discoverable logical foundations, which will show that the use of the Feynman integral entails contradictions. No quantum physicist in his right mind would pay the slightest attention to such logical hair-splitting (cf., also Schwartz, 2001, Chapter VI).

I am suggesting, therefore, that a sympathetic reader (an always elusive creature), should approach this famous dialogue between a great philosopher of, among other things, mathematics, and a great logician and founding father of computability theory, remembering that both of these intellectual giants were also, fundamentally, wedded to the 'physical world' – one with an explicit engineering background and the other as an 'applied mathematician', both camouflaging as logicians and mathematicians perplexed by semantic paradoxes and grammatical nuances that they thought could be sorted out by dialogue.

---

[6] If, at this point, Ulam had added '*and act*', he would have completely encapsulated Wittgenstein's prescription for circumventing contradictions in mathematics and logic by means of '*rules*'.

[7] The original journal article has 'motions', but the context makes it clear that what is meant is 'notions'.

[8] Dirac himself attributed the origin of the idea to his 'early engineering training' (cf., Kragh, 1990, p.41) – surely paralleling both Wittgenstein's early training as an aeronautical engineer and Turing's above characterisation by Newman and Hodges.

References

Brouwer, Luitzen. E.J (1913), *Intuitionism and Formalism*, **Bulletin of the American Mathematical Society**, Vol. 20, #2, pp. 81-96.

Chihara, Charles (1977), *Wittgenstein's Analysis of the Paradoxes in His Lectures on the Foundations of Mathematics*, **The Philosophical Review**, Vol. 86, # 3, July, pp. 365-381.

Cooper, S. Barry, Benedikt Löwe & Andrea Sorbi (2008), **New Computational Paradigms: Changing Conceptions of What is Computable**, Springer Science+Business Media, LLC, New York.

Feferman, Solomon (1991), *Preface* to: *Systems of Logic Based on Ordinals*, in: **Collected Wroks of A.M. Turing – Mathematical Logic**, edited by R. O. Gandy & C. E. M Yates, North-Holland, Amsterdam

Hodges, Andrew (2008), *Alan Turing, Logical and Physical*, pp. 3-15, in: Cooper, et.al., edited (2008).

Kragh, Helge. S (1990), **Dirac: A Scientific Biography**, Cambridge University Press, Cambridge.

McCloskey, D. N (1991), *Economic Science: A Search Through the Hyperspace of Assumptions?*, **Methodus**, Vol. 3, # 1, June, pp. 6-16.

Rota, Gian-Carlo (1986), *In memoriam of Stan Ulam: The Barrier of Meaning*, **Physica 22D**, pp. 1-3.

Schwartz, Lauretnt (2001), **A Mathematician Grappling with His Century**, Birkhäuser Verlag, Basel.

Shanker, Stuart. G (1987), **Wittgenstein and the Turning-Point in the Philosophy of Mathematics**, State University of New York Press, Albany, NY.

Wittgenstein, Ludwig (1939 [1976]), **Wittgenstein's Lectures on the Foundations of Mathematics – Cambridge 1939,** edited by Cora Diamond, The University of Chicago Press, Chicago.

II. Turing on *Solvable and Unsolvable Problems* & Simon on *Human Problem Solving*[9]

Turing's fundamental work on *Solvable and Unsolvable Problems* (Turing, 1954), *Intelligent Machinery* (Turing, 1969) and *Computing Machinery and Intelligence* (Turing, 1950) had a profound effect on the work of Herbert Simon, the only man to win both the *ACM Turing Prize* and the *Nobel Memorial Prize in Economics*, particularly in defining *boundedly rational* economic agents as *information processing systems* (IPS) solving decision problems[10].

A comparison of Turing's classic formulation of *Solvable and Unsolvable Problems* and Simon's variation on that theme, as *Human Problem Solving* (Newell & Simon, 1972), would be an interesting exercise, but it must be left for a different occasion. This is partly because the *human problem solver* in the world of Simon needs to be defined in the same way Turing's approach to *Solvable and Unsolvable Problems* was built on the foundations he had established in his classic of 1936-37.

It is little realised that four of what I call the *Five Turing Classics*[11] – *On Computable Numbers* (Turing 1936-7), *Systems of Logic* (Turing, 1939), *Computing Machinery and Intelligence* and *Solvable and Unsolvable Problems* – should be read together to glean *Turing's Philosophy*[12] *of Mind*. Simon, as one of the acknowledged founding fathers of

---

[9] Due to severe space constraints I am forced, most regrettable, to omit the legitimate place that should be accorded the classic study by Adriaan De Groot on ***Thought and Choice in Chess*** (De Groot, 1965 [1978]). Herbert Simon's warm endorsement of this classic study by De Groot, on the dustjacket of the second edition of the book is worth reporting here:

> "Professor de Groot's book is an important contribution to the data and theory of *human problem solving* that is particularly remarkable in the way in which it ties together many strands of intellectual inquiry, previously so widely separated in time and place.

De Groot was fully aware of Turing's contribution to a problem solving approach to playing games, particularly chess (op.cit, p. 24 and, in particular, footnote 19), referring to Turing (1953).

[10] In the precise sense in which this is given content in mathematical logic, metamathematics, computability theory and model theory.

[11] The fifth being, of course, ***The Chemical Basis of Morphogenesis*** (1952). It is interesting to note that the five contributions came in two clusters, the first two in 1936-7 & 1938/9; the last three in the fertile last four years of his tragically brief life.

[12] Remembering Feferman's (1991, p. 79) cautionary note that Turing never tried to develop an over-all *philosophy of mathematics* …', but not forgetting that his above works were decisive in the resurrection of a particular vein of research in the philosophy of mind, particularly in its cognitive, neuroscientific, versions pioneered by Simon.

computational cognitive science was deeply indebted to Turing in the way he tried to fashion what I have called *Computable Economics* (Velupillai, 2000)[13]. It was not for nothing that Simon warmly acknowledged – and admonished – in his essay in a volume 'memorializing Turing' (Simon, 1996, p. 81), titled *Machine as Mind*[14]:

> "If we hurry, we can catch up to Turing on the path he pointed out to us so many years ago."

Simon was on that path, for almost the whole of his research life.

Building a Brian, in the context of economic decision making, meant a mechanism for encapsulating human intelligence, underpinned by rational behaviour in economic contexts. This was successfully achieved by Herbert Simon's lifelong research program on computational behavioural economics[15].

From the early 1950s Simon had empirically investigated evidence on human problem solving and had organised that evidence within an explicit framework of a theory of sequential information processing by a Turing Machine. This resulted in (Simon, 1979, p. x; italics added):

> "[A] general theory of human cognition, not limited to problem solving, [and] a methodology for expressing *theories of cognition as programs* [for digital computers] and for using [digital] computers [in general, Turing Machines] to simulate *human thinking*.

This was the first step in replacing the traditional Rational Economic Man with the computationally underpinned Thinking i.e., Intelligent - Man. The next step was to stress two empirical facts (ibid, p. x; italics added):

---

[13] I could as well have called it *Turing's Economics*.

[14] To which he added the caveat (ibid, p. 81):

> "I speak of 'mind' and not 'brain'. By mind I mean a system [a mechanism] that produces thought .. .'

I have always interpreted this notion of 'mechanism' with Gandy's *Principles for Mechanisms* (Gandy, 1980) in *mind* (sic!).

[15] I refer to this variant of behavioural economics, which is underpinned by a basis in *computational complexity theory*, as *classical* behavioural economics, to distinguish it from currently orthodox behavioural economics, sometimes referred to as *modern* behavioural economics, which has no computational basis whatsoever.

i. ‘There exists *a basic repertory of mechanisms and processes* that Thinking Man uses in all the domains in which he exhibits *intelligent behaviour*.”

ii. “The models we build initially for the several domains must all be assembled from this same basic repertory, and common principles of architecture must be followed throughout.”

It is easy to substantiate the claim that the *basic repertory of mechanisms and processes* are those that define, in the limit, a Turing Machine formalisation of the Intelligent Man, when placed in the decision-making, problem-solving, context of economics (cf. Velupillai, 2010).

The broad contours of this vision and method, and its basis in computability and computational complexity theory, were clearly outlined in a letter he wrote me, after reading my book on *Computable Economics* (Simon, 2000):

> “I want to share some first impressions on my reading of “Computable Economics.” … I was delighted and impressed by the mileage you could make with Turing Computability in showing how nonsensical the Arrow/Debreu formulation, and others like it, are as bases for notions of human rationality.
>
> As the book makes clear, my own journey through bounded rationality has taken a somewhat different path. Let me put it this way. There are many levels of complexity in problems, and corresponding boundaries between them. *Turing computability is an outer boundary, and as you show, any theory that requires more power than that surely is irrelevant to any useful definition of human rationality*. A slightly stricter boundary is posed by computational complexity, especially in its common “worst case” form. We cannot expect people (and/or computers) to find exact solutions for large problems in computationally complex domains. This still leaves us far beyond what people and computers actually CAN do. The next boundary, but one for which we have few results …, is computational complexity for the “average case”, sometimes with an “almost everywhere” loophole. That begins to bring us closer to the realities of real-world and real-time computation. Finally, we get to the empirical boundary, measured by laboratory experiments on humans and by observation, of the level of complexity that humans actually can handle, with and without their computers, and - perhaps more important – what they actually do to solve problems that lie beyond this strict boundary even though they are within some of the broader limits.
>
> The latter is an important point for economics, because we humans spend most of our lives making decisions that are far beyond any of the levels of complexity we can handle exactly; and this is where satisficing, floating aspiration levels, recognition and heuristic search, and similar devices for arriving at good-enough decisions take over. A parsimonious economic theory, and an empirically verifiable one, shows how human beings, using very simple procedures, reach decisions that lie far beyond their capacity for finding exact solutions by the usual maximizing criteria.
>
> ….
>
> So I think we will continue to proceed on parallel, but somewhat distinct, paths for examining the implications of computational limits for rationality – you the

> path of mathematical theories of computation, I the path of learning how people in fact cope with their computational limits
> ….
> While I am fighting on a somewhat different front, I find it greatly comforting that these outer ramparts of Turing computability are strongly manned, greatly cushioning the assault on the inner lines of empirical computability.”

Unfortunately, the ‘assaults’ by orthodoxy and its non-computable, non-constructive forces are ceaseless and ‘cushioning’ the ‘inner lines of empirical computability’ from these persistent assaults is no easy task.

References

De Groot, Adriaan D (1965 [1978]), **Thought and Choice in Chess** (Second Edition), Mouton Publishers, The Hague.

Gandy, Robin. O (1980), Church's Thesis and Principles for Mechanisms, in: The Kleene Symposium, edited by J. Barwise, H. J. Keisler and K. Kunen, North-Holland, Amsterdam.

Feferman, Solomon, (1991), *Preface* to: *Systems of Logic Based on Ordinals*, in: **Collected Wroks of A.M. Turing – Mathematical Logic**, edited by R. O. Gandy & C. E. M Yates, North-Holland, Amsterdam

Newell, Allen & Herbert A. Simon (1972), **Human Problem Solving**, Prentice-Hall Inc., Englewood Cliffs, NJ.

Simon, Herbert. A (1979), **Models of Thought: Volume I**, Yale University Press, New Haven.

Simon, Herbert. A (1996), *Machine as Mind*, Chapter 5, pp. 81-101, in: **Machines and Thought - The Legacy of Alan Turing**, *Volume 1*, edited by Peter Macmillan and Andy Clark, Oxford University Press, Oxford.

Simon, Herbert. A (2000), *Letter to Velupillai*, 25 May, 2000.

Turing, Alan M (1953), *Digital Computers Applied to Games*, Chapter 25, pp. 286-310, in: **Faster than Thought**, edited by B. V. Bowden, Pitman, London.

Velupillai, K. Vela (2000), **Computable Economics**, Oxford University Press, Oxford.

Velupillai, K. Vela (2010), *Foundations of Boundedly Rational Choice and Satisfying Decisions*, **Advances in Decision Sciences**, April.

III. Four Traditions of *Emergence*: Morphogenesis, Ulam-von Neumann Cellular Automatas, the Fermi-Pasta-Ulam Problem and British *Emergentism*

The classic works by the trio of John Stuart Mill (1890), George Henry Lewes (1891) and C. Lloyd Morgan (1927), together with C. D. Broad (1929) and Samuel Alexander (1920) made up what has come to be called the *'British Emergentist'* school. The concept of *emergence* came to have its current connotations as a result of these (and a few other) clearly identifiable sequence of classic works by these pioneering British philosophers. A representative sample of crucial definitions, in these classics, may provide a decent backdrop against which to proceed.In particular, the one by Lewes, the man who introduced the word *'emergent'*, from which Lloyd Morgan derived *'emergence'*:

> “[T]here are laws which, like those of chemistry and physiology, owe their existence to .. *heteropathic laws*… . The Laws of Life will never be deducible from the mere laws of the ingredients, but the prodigiously *complex* Facts of Life may all be deducible from comparatively *simple* laws of life;…”
> John Stuart Mill (1890) Bk.III, Ch.VI, p.269; italics added.
>
> “Thus, although each effect is the resultant of its components, the product of its factors, we cannot always trace *the steps of the process*, so as to see in the product the mode of operation of each factor. In this latter case, I propose to call the effect *an emergent*. It arises out of the combined agencies, but in a form which does not display the agents in action.”
> George Henry Lewes (1891) Problem V, Ch.III, p.368, italics added.
>
> “The concept of *emergence* was dealt with .. by J.S.Mill in his Logic .. under the discussion of *`heteropathic laws'* in causation. The word *`emergent'* as contrasted with *`resultant,'* was suggested by G.H.Lewes in his Problems of Life and Mind'. … . What makes emergents emerge? .. *What need [is there] for a directive Source of emergence. Why should it not proceed without one?”*
> C. Lloyd Morgan (1927), pp. 2, 32; italics added.

The rise and fall of *British Emergentism* has been eloquently and almost persuasively argued by Brian McLaughlin (1992), basing himself on the emerging (sic!) codification of quantum mechanics in the works of Heisenberg, Schrödinger, Dirac and Pauling, and on the philosophical critiques of the 1920s, launched primarily by Stephen Pepper (1926), W. T. Stace (1939) and Charles Bayliss (1929).

Remarkably, it is possible, with a good dose of hindsight, to pinpoint the reason for McLaughlin's premature obituary of *British Emergentism* and, at the same time, link that failure with a prescient – and typically penetrating, yet almost playfully formulated – observation by Alan Turing in his last publication before a tragically truncated life came to an end.

In what can only be called a moment of weakness, because Dirac, surely, is not capable of carelessness or flippancy, one of the great founding fathers of modern quantum mechanics, slipped badly in a pronouncement which was the fulcrum around which the premature obituary of *British Emergentism* was proclaimed[16]:

> "The underlying physical laws necessary for the mathematical theory of a large part of physics and *the whole of chemistry are thus completely known*[17], and the difficulty is only that the exact application of these laws leads to equations much too complicated to be soluble. It therefore becomes desirable that approximate practical methods of applying quantum mechanics should be developed, which can lead to an explanation of the main features of complex atomic systems *without too much computation*."
> Dirac (1929), p. 714; italics added.

Contrast this with Turing's wonderfully laconic, yet eminently sensible precept (Turing, 1954, p.9; italics added):

[16] It was little realised, at the time such views were invoked to pronounce the premature death of *British Emergentism*, that Dirac had attended Broad's lectures at Bristol University, before both he and Broad moved to Cambridge – and, even more importantly, that Dirac had read, with customary care, the whole of Mills' **A System of Logic** (cf. Farmelo (2009), in particular, Chapter 3).

[17] This must rank with the celebrated but, mercifully, falsified prophetic pontifications by two other intellectual giants of the 19th century: Lord Kelvin and John Stuart Mill. The former is reputed to have *suggested*, on the eve of the works by Planck and Einstein, that *all the problems of physics had been solved*', except for just two anomalies: the Michelson-Morley experiment, on the one hand, and Black Body radiation, on the other'! As for the great and saintly John Stuart Mill, in what can only be called an unfortunate moment of weakness, he etched for posterity theese un-prophetic thoughts (Mill, 1848, [1898], Bk. III, Ch. 1, p. 266; italics added):

> "*Happily, there is nothing in the laws of Value which remains for the present writer to clear up; the theory of the subject is complete*: the only difficulty to be overcome is that of so stating it as to solve by anticipation the chief perplexities which occur in applying it: and to do this, some minuteness of exposition, and considerable demands on the patience of the reader, are inevitable."

> "No mathematical method can be *useful* for *any* problem *if it involves much calculation*."

It is precisely in this sense that Eric Scerri (1994) – and in a series of perceptive writings on the failure of reductionism, in general, and the untenability of McLaughlin's thesis (Scerri & McIntyre, 1997, Scerri, 2007) – has made his case against Dirac's unfortunate claim. The *British Emergentists* were prescient in their approach to the formalization of emergence, coupled to the dialectic between the simple and the complex, in a natural dynamic context. They *rise and rise*; there was **never any fall** of the *British Emergentisits*!

Turing's remarkably original work on *The Chemical Basis of Morphogenesis* was neither inspired by, nor influenced any later allegiance to the British Emergentist's tradition – such as the influential experimental and theoretical neurological and neurophilosophical work of Nobel Laureate, Roger Sperry[18].

On the other hand, the structure of the experimental framework Turing chose to construct was uncannily similar to the one devised by Fermi, Pasta and Ulam, (1955), although with different purposes in mind. But there was – and there remains – a deeper affinity in that the violation of the equipartition of energy principle that was observed in the Fermi-Pasta-Ulam simulation and the symmetry-breaking that is intrinsic to the dynamical system behaviour of Turing's system of reaction-diffusion equations.

Turing's aim was to devise a mechanism by which a spatially homogeneous distribution of chemicals – i.e., formless or patternless structure - could give rise to form or patterns via what has come to be called a *Turing Bifurcation*. A reaction-diffusion mechanism formalised as a (linear) dynamical system and subject to what I have referred to, in other writings, as *the linear mouse theory of self-organisation*[19].

---

[18] See, in particular, Sperry's outstanding Noble Prize Lecture, delivered on 8 December, 1981,on the nature of the emergence of consciousness and its relation to brain processing.

[19] In typically playful fashion, he summarised the mathematical mechanism he sought (Turing, 1952, pp. 43-4):

> "Unstable equilibrium is not … a condition which occurs very naturally. .. Since sytems tend to leave unstable equilibria they cannot often be in them. *Such equilibria can, however, occur naturally through a stable equilibrium changing into an unstable one*. For example, if a rod is hanging from a point a little above its centre of gravity it will be in stable equilibrium. If, however, a mouse climbs up the rod the equilibrium eventually becomes unstable and the rod

As young boy, Alan Turing won the *Morecom science prize*[20] for his work on the study of 'the reaction between *iodic acid* and sulphur dioxide' (cf., Hodges, 1983, p. 52; italic added). Indeed, even as a twelve year old boy, during Christmas holidays spent at the family villa in the Rue du Casino[21], Alan Turing had 'heaved great quantities of sea-weed .. from the beach in order to extract a minute amount of iodine' (ibid, p. 18). It is, therefore, particularly satisfying to note that the kind of patterns suggested by Turing's theory of morphogenesis was first definitively established in *iodine reactions,* in the work of Castets, et.al., (1990) and Ouyang & Swinney (1991). How much more serendipitous could events be?

In this same vein, it is most satisfying to note the role the *Turing Bifurcation* played in the development of the *Brusselator* and the work of the 1977 Chemistry Nobel Prize winner, Ilya Prigogine (cf. Nicoils and Prigogine, 1977) on self-organisation in non-equilibrium systems.

I have come to try to characterise, at least for the purpose of classifying in some systematic way, the contributions to emergence in terms of: (i). Novelty; (ii). Irreducibility; (iii). Unpredictability; (iv). Non-reductive Physicalism; (v). Downward Causation.

These are the categories that played decisive roles in the emergence literature that originated in the work of the British Emergentists. Perhaps the time is apposite for a reconsideration of the philosophical underpinnings of Turing's methodology for morphogenesis. If so, then it is the basis in the work of the British Emergentists, and their above characterising categories, that one may find the way forward. This will not be incongruent at all, given that Lloyd Morgon was, among other things, also a zoologist, a pupil of T.H. Huxley and the man who coined the word Emergence in his famous *Gifford Lectures*. Add to this the names of Sperry and Prigogine, and the trio of Fermi, Pasta and Ulam and their experimental structure, and it would be a simple completion of an honour roll when Turing's name is added to the list – and this, too, on the basis of only one of his many fundamental contributions.

> starts to swing. … The system which was originally discussed … might be supposed to correspond to the mouse somehow reaching the top of the pendulum without disaster, perhaps by falling vertically on to it."

[20] Christopher Morecom was Alan's dear friend during the very brief period they shared at Sherborne school and the Morecom family, on the unfortunately early death of their son, had endowed, in memory of their son, 'a science prize to be awarded for work which included an element of originality', (Hodges, op.cit., p. 51).

[21] In Dinard, France.

I have also found it useful to utilise the following three precise notions for the classifying exercise: *Potential Surprise*, *Computation Universality*, *Mereological Confusion*. In every one of the five classifying categories and the three analytical notions used for the classifying exercise, I have been inspired by some aspect of Turing's work.

Above all, it is by now only too well known that von Neumann's contribution – in his famous joint work with Ulam – to the theory of self-reproducing automata, was almost wholly underpinned by Turing's theory of computation.

References

Alexander, S. (1920), **Space, Time and Deity- The Gifford Lectures at Glasgow**, *Vol. I & Vol. II*. London: Macmillan and Co.

Bayliss, C.A. (1929), *The Philosophic Functions of Emergence*, **The Philosophical Review**, Vol. 38, # 4: 372 - 384.

Broad, C.D. (1929), **The Mind and its Place in Nature,** *The Tarner Lectures delivered at Trinity College, Cambridge, 1923*. Kegan Paul, Trench. London: Trubner & Co., Ltd.

Castets.V, E. Dulos, J. Boissonade, and P. De Kepper (1990), *Experimental evidence of a sustained standing Turing-type nonequilibrium chemical pattern*, **Physical Review Letters**, Vol. 64, # 24, pp. 2953-2956.

Dirac, P. A. M (1929), *Quantum Mechanics of Many-Electron Systems*, **Proceedings of the Royal Society of London, Series A**, Vol. 123, # 792, 6 April, pp. 714-733.

Farmelo, Graham (2009), **The Strangest Man: The Hidden Life of Paul Dirac, Quantum Genius**, Faber and Faber, ltd., London.

Fermi, E., Pasta, J., and Ulam, S, (1955), *Studies of Non Linear Problems*, Los Alamos Preprint, LA-1940, May.

Hodges, Andrew (1983), **Alan Turing: The Enigma**, Burnett Books, London.

Lewes, G.H. (1891), **Problems of Life and Mind**. New York: Houghton, Miffin & Co.

Lloyd Morgan, C (1927) **Emergent Evolution**, *The Gifford Lectures* (2nd Edition). London: Williams & Norgate.

McLaughlin, B.P. (1992) *The Rise and Fall of British Emergentism* in: **Emergence or Reduction - Essays on the Prospects of Nonreductive Physicalism**. edited by Ansgar Beckermann, Hans Flohr & Jaegwon Kim (pp. 49-93). Berlin: Walter de Gruyter.

Mill, John Stuart (1848; [1898]), **Principles of Political Economy with Some of Their Applications to Social Philosophy**, Longmans, Green, and Co., London.

Mill, John Stuart (1890), **A System of Logic** (8th Edition). New York: Harper & Brothers Publishers.

Nicolis, G & Ilya Prigogine (1977), **Self-Organization in Non-Equilibrium Systems**, John Wiley & Sons, New York.

Ouyang, Q & H.L Swinney (1991), *Transition from a Uniform State to Hexagonal and Striped Turing Patterns*, **Nature**, vol. 352, pp. 610-12.

Pepper, S. C (1926) *Emergence*, **The Journal of Philosophy**. Vol. 23, # 9: 241 - 245.

Scerri, Eric R (1994), *Has Chemistry Been at Least Approximately Reduced to Quantum Mechanics*, **Proceedings of the Biennial Meeting of the Philosophy of Science Association**, Volume One: Contributed Papers, pp. 160-170.

Scerri, Eric. R & Lee McIntyre (1997), *The Case for the Philosophy of Chemistry*, **Synthese**, Vol. 111, # 3, June, pp. 213-232.

Scerri, Eric. R (2007), *Reduction and Emergence in Chemistry – Two Recent Approaches*, **Philosophy of Science**, Vol. 74, #5, pp. 920-931.

Sperry, R.W (1981), *Some Effects of Disconnecting the Cerebral Hemispheres*, **Nobel Prize Lecture**, Stockholm, 8 December, 1981.

Stace, W.T. (1939), *Novelty, Indeterminism and Emergence*, **The Philosophical Review**. Vol. 48, #3: 296 - 310.

Turing, Alan. M (1954), *Solvable and Unsolvable Problems*, **Science News**, #31, pp. 7-23.

von Neumann, J. (1966), **Theory of Self-Reproducing Automata**, edited and completed by Arthur W. Burks, University of Illinois Press, Urbana.