Stabilisation Policy – Phillips before the ‘Phillips Curve’

KATHERINE MOOS & K. VELA VELUPILLAI
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Katherine Moos & K. Vela Velupillai*
Department of Economics
The New School for Social Research (NSSR)
New York
USA

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“Phillips wrote down the relation that subsequently became famous as the ‘Phillips curve’ and built it into his early models in order to remove [the classical dichotomy]. ... Although Phillips regarded the shedding of the dichotomy as crucial to the development of useful models of stabilization policy, he did not think it important to go further than just to write down a relation that captured what to him seems so obvious. It is perhaps ironic that the world-wide fame of his name as well as his enormous influence over a decade of economic research stems to a great extent from what he regarded as both minor and an obvious (although critical) part of his system.

The now famous curve made its debut in 1954 in Phillips’ first major published paper, ‘Stabilization* Policy in a Closed Economy’.”


* This is a contribution to commemorate the 60th anniversary of the first of two pioneering contributions by A.W. Phillips (1954), to a dynamic analysis of what was then referred to as Stabilisation Policy, within a Keynesian (Closed, Linear) Multiplier-Accelerator framework. A more integrated and relatively complete analysis, taking into account both the 1954 classic and the subsequent Phillips (1957) extension, is the subject of a paper under preparation by the authors, set within the broader context of the contemporary – i.e., early 1950s – literature on the Theory of Economic Policy (stemming, mostly, from the Scandinavian and Dutch discussion of the issues in the interface between theory and policy).

* Velupillai is, simultaneously, Professor of Economics and the Director of the Algorithmic Social Sciences Research Unit (ASSRU), in the department of economics at the University of Trento, Trento, Italy. He dedicates this paper to his two beloved teachers, Björn Thalberg in Lund, and Richard Goodwin, in Cambridge, both of whom introduced him to the Phillips Machine and the two stabilization policy classics by Phillips, over forty years ago. It is a twist of fate that Thalberg died almost exactly on the one hundredth anniversary of Goodwin’s birth!

* The spelling in Phillips (1954) is in terms of ‘British’ English, i.e., Stabilisation, rather than ‘stabilization’; we have followed the Phillips practice in this paper.
§1. The Setting

“In [Phillips (1954)], a remarkable essay that emerged from his dissertation, Bill Phillips demonstrated that automatic stabilization policies can exacerbate instability rather than contain it. … The essay also offers an intuitive description of one process that can be taken to underlie the Phillips Curve. In addition such a graph is drawn in the paper, though it is based neither on empirical data nor on any formal model.”

Baumol, 2000, p. 282; italics added.

The ‘graph’ Baumol refers to, which ‘is based neither on empirical data nor on any formal model’ appears as fig. 11 (p. 308 – reproduced here as Fig. 1, for convenience – in Phillips, 1954);

![Graph](image)

Fig. 1

It is to this that Lipsey (op.cit) refers as the now famous [Phillips] curve [which] made its debut in 1954 in …, ‘Stabilization Policy in a Closed Economy’. It is at least of doctrine historical interest to recall the guarded way Phillips qualified the form of the graph:

“We may therefore postulate a relationship between the level of production and the rate of change of factor prices, which is probably of the form shown in [Fig 1], the fairly sharp bend in the curve where it passes through zero rate of change of prices being the result of the greater rigidity of factor prices in the downward direction. … In spite of the marked curvature of the relationship, linearity may be assumed as approximation for small changes in production.”

1 Perceptive and knowledgeable readers will recall Fig.1 in Goodwin (1967, p. 55) and his assumption of linearity of the relationship between the proportional rate of change of real wages and the unemployment ratio. This kind of postulate goes back not just to Irving Fisher, as many have observed, but all the way to Marx – and even Ricardo!
The relationship that was made ‘famous’ in Phillips (1958) was, of course, between unemployment rates and the rate of change of money wages – but the idea of the form of the ‘graph’ remained, albeit in a mirror reflected form.

The bare fact of the provenance of the classic of 1954 is stated in the opening sentence of the lead footnote in the paper (footnote 1, p.290):

“This article is based on part of the material of a thesis submitted to the University of London for the degree of Ph.D.”

The Ph.D thesis was titled Dynamic Models in Economics, examined by Sir John Hicks on 10 December, 1953, with the degree itself awarded on 27 January, 1954 – which makes this 60th anniversary homage doubly apposite.

Unlike Lipsey, however, our conjecture is that the curved form of the graph had to be postulated by a Phillips who was simulating a system of differential equations by an analogue computing device of his own construction – the Phillips Machine. Phillips’ simulations in Phillips (1954, 1957) were not those of an approximation or a discretization of the system of theoretical differential equations he had set up. They were exact simulations of a system evolving according to linear differential equations (not liner difference- differential equations, nor linearized nonlinear differential equations).

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2 It is interesting to remember that Phillips (1950) was titled: Mechanical Models in Economic Dynamics. Surely the addition of the word Mechanical to this title reflected the fact that the article was devoted, almost exclusively to a descriptive and engineering explanation of the design and structure of the Phillips Machine, but resting on the dynamic economic foundation of Keynesian macroeconomics built on multiplier-accelerator bases. This is most evident in Phillips (1950, p.289, footnote 4), where the reliance on Goodwin (1948) is made clear, but the first reference to the ‘Hicks-Goodwin theory of limit cycles in a non-linear system’ is in his review (Phillips, 1954a, p. 364), of Kalecki (1954), in the same year as the classic of 1954 – which makes this a triple 60th anniversary homage! Goodwin’s (1956) caustic review of this (mostly unrevised) collection of previously published essays by Kalecki is written in the typical maestro’s style of ‘Let’s carve him as a dish fit for the gods, Not hew him as a carcass fit for hounds’ (pace, Caesar, Act 2, Sc. 1).

3 Not that we disagree with Lipsey’s explanation in terms of the attempt by Phillips to remove the sharpness of the classical dichotomy; it is just that the engineer in Phillips, with the analogue Machine he constructed, at his disposal, could not possibly make the hydraulic and mechanical system function without ‘breaking’ the ‘dichotomy’ and making it a ‘smooth curve’.
To the best of our knowledge, no one\(^4\) has emphasized this particular point: that all the simulations reported in Phillips (1954, 1957) are those of exact solutions, implemented not on a digital computer, but (presumably) on an analogue device, most likely the Phillips Machine\(^5\).

The macroeconomic underpinnings of the models in Phillips (op.cit) are of the traditional multiplier-accelerator form, for one closed macroeconomy, albeit in their linear variants (although the inspirations are obtained from the nonlinear Goodwin models of 1948-1951; this is particularly evident in Phillips (1950)).

This brief homage to the 60\(^{\text{th}}\) anniversary of Phillips’ pioneering contribution to stabilisation policy is, therefore, structured as follows. In the next section we outline the underpinnings of the basic framework, mathematical and economic, in Phillips (1954). In § 3, we attempt to suggest three extensions – nonlinearity, coupled macroeconomies and the resulting inevitable study of coupled nonlinear systems by simulations (simply because closed, computably analytic solutions of such systems are provably (almost) impossible to derive.

§2. A Discussion

“The structure of macro-economic models is shown up very clearly in terms of schematic or block diagrams, developed by engineers and first used for economic systems by Phillips [1954]\(^6\].”
Allen, 1967, p. 97; bold phrase in the original; italics added.

The bare bones of the underpinning economics of the model of Stabilisation Policy in a Closed Economy, is that of an expenditure-based, multiplier-accelerator dynamics, of a closed, aggregative, macroeconomy, of the then accepted framework of Keynesian economics. The later much misunderstood and narrow (very American) distinction between ‘Keynesian Economics’ and the ‘Economics of Keynes’ had not, as yet, crystallised – after all Samuelson’s acknowledged misnomer, the neoclassical synthesis, had only appeared in the 1953 edition of his

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\(^4\) With the possible exception of Thalberg (1971a, 1971b); see the opening quote in § 3.

\(^5\) Interestingly, there is no particular reference to the Phillips Machine in Phillips (1954) – perhaps because the wholly linear systems considered in this paper are all exactly solvable.

\(^6\) This is, strictly speaking, not correct. Goodwin used block diagrams most felicitously, already in 1951 (see, Goodwin (1951), p. 7).
popular Economics textbook (a phrase coined, according to Samuelson, ‘to get McCarthy off [his] back’). However, the multiplier-accelerator dynamics is, itself, linear.

Thus the mathematical formulations of the economic models are in terms of linear, ordinary, differential equations – at most of order four, but ranging from two to four, depending on the various combination of proportional, derivative and integral policies that are integrated into the underlying multiplier-accelerator models.

The pure – i.e., without the addition of proportional, derivative or integral policies, or any combinations of them – multiplier-accelerator model⁷ (equation 7e, p. 321) is a linear, second-order, ordinary differential equation; this model, with the sole addition of proportional stabilization policies becomes a linear, third-order, ordinary differential equation (equation 7f, p. 321); finally, the multiplier-accelerator model with either proportional plus integral stabilization policy regimes, or all three policy regimes together, becomes a fourth-order, ordinary differential equation (equations 7g, p. 321 or 7h, p. 322).

Determining the initial conditions for the higher order equations, in the context of Phillips’ fertile use of a background block-diagram structure for the various systems (see op.cit, Fig. 12, p. 309), are in terms of simple rules for systematic and sequential integration, as set out in footnote 1 of pp. 319-320.

The simplicity of these rules can be shown by considering the block diagram for a second-order linear differential equation, and the rules for systematic and sequential integration. Consider the linear, second order, differential equation that once formed the fountainhead of Keynesian, endogenous, macroeconomic theories of the cycle (the canonical equation in the Phillips differential equation model of the macroeconomy, 7e):

\[ ad^2x/dt^2 + bdx/dt + kx = F \]  

(1)

where, of course, a, b, c and F are given parameters;

⁷ In which price and interest rate variations are abstracted from and, in any case, when changes in these crucial monetary variables are explicitly considered in Phillips (1954), they are done so for the pure multiplier model, ‘excluding the acceleration relationship’ (op.cit., p. 322).
Solving for the second order term, $d^2x/dt^2$:

$$d^2x/dt^2 = (1/a)F - (k/a) \frac{dx}{dt} - (b/a)x \quad (2)$$

Integrating (2) gives the value for $dx/dt$ to be replaced in the third term in the above equation and integrating $x$, gives the value for $x$, and the system is ‘solved’.

It should be remembered that these are exact integrations, given the linearity of the system, and closed form solutions are obtained for the given initial conditions.

Thus, three mechanical, electrical or even hydraulic elements have to be put together in the form of a block diagram, eventually to form an analogue computing machine such as the one Phillips constructed in 1949, to implement a solution for (2):

- An element that would add terms, denoted by a circle;
- An element that could multiply constants or variables by constants, denoted by an equilateral triangle;
- An element that could `integrate', in the formal mathematical sense, without resorting to sums and limiting processes, denoted by a ‘funnel-like’ symbol;

Thus, three circles, one equilateral triangle and two ‘funnels’, connected as in Figure 2, can solve the above equation:

![Block diagram for a second-order, ordinary, differential equation.](image)

Fig. 2: Block diagram for a second-order, ordinary, differential equation.
One final remark is necessary here. Although we have highlighted – and Thalberg (1971b) emphasized – the simulation aspect of the stabilization policy exercises carried out in Phillips (1954), it is, surely, clear that exact, closed form, analytic solutions are feasible, given the initial conditions, for all of the differential equations considered by Phillips. In this sense it is not clear that any meaningful simulation is required, to learn about the characterization of the attractors of the dynamical systems and, thus, to determine the trajectories towards the determined equilibria.

The various policy regimes can, in fact, be considered ‘proxies’ for ‘external shocks’, in the terminology of currently fashionable orthodoxies, although, of course, these proportional, derivative and integral – and combinations thereof – alternatives are not ‘stochastically’ determined. They are part of an exploration of alternative dynamics and, in this sense are a rich and copiously imaginative exercise in structured simulations of a variety of feasible and desirable policy regimes.

In this sense the exercise in Phillips (1954) is outside the formal theory of economic policy – either in its classic target-instrument approach, or in its more modern, mindless, optimal control (or dynamic programming) determined policy nihilistic exercises.

We like to interpret it as the setting stage for the extensions that were mentioned, ‘here and there’, in the various Phillips contributions from 1949 to the eve of the (in)famous advent of the Phillips Curve. This irreversibly changed the course of policy oriented macroeconomics, first by emphasizing trade-off between inflation and unemployment, in the context of macroeconomic dynamics, then by the eternal new dichotomies between short and long term dynamics, adaptive and rational expectations and, finally, the ultimate in policy nihilism, variations of the Lucas critique, culminating in a poor imitation of the richness in Phillips (1954) by the overwhelming emphasis on Taylor rules, for a narrow and jaundiced approach to monetary policy.
The rich and varied tradition of a theory of economic policy where there was no question of a trade-off between inflation and unemployment, because there was an underpinning in relevant institutional economics, was all but forgotten.

§3. An Extension – or Three

“In two well-known articles .. in The Economic Journal in 1954 and 1957, A. W. Phillips illustrated some general problems of stabilization policy with the help of a few simple dynamic models and simulation techniques. … However, the adequacy of Phillips’ methods may be questioned. Phillips .. stressed the need for further studies. Following his specific suggestions, the author seeks to extend Phillips’ analysis by incorporating non-linearities, growth trend and multiple objectives in a simple way.”
Thalberg, 1972b, p. 385; second set of italics, added.

Three obvious extensions are in order:

1. A nonlinear multiplier-accelerator system of Keynesian macrodynamics;
2. A consideration of open macroeconomics;
3. A framework for structured simulations;

A consideration of the first two extensions together implies, in an endogenous macroeconomic framework, coupled dynamics. This almost necessarily involves a consideration of structured simulations simply because coupled nonlinear systems of differential equations are rarely amenable to closed form analytic solutions. Phillips was well aware of the feasibility of the first two extensions within the broad framework given in Phillips (1950) and Phillips (1954). For example in the former, he states very explicitly, (pp. 287-288; italics added)8:

“The hydraulic model will give solutions for non-linear systems as easily as for liner ones. It is not even necessary for the relationship to be in analytic form: so long as the curves can be drawn9 the machine will record the correct solutions, within the limits of its accuracy. In giving the equivalent mathematical model, however, the usual linearity

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8 As Goodwin (1950, p. 319, footnote 6) picturesquely, if also soberly, noted:
“Combining the difficulties of difference equations with those of non-linear theory, we get an animal of a ferocious character and it is wise not to place too much confidence in our conclusions as to behavior.”

9 See McRobie (2010) and Goodwin (2000).
assumption will be made, in view of the difficulty of working with non-linear differential or difference equations.”

Similarly, in relation to the initially restrictive assumption of a closed macroeconomy, in Phillips (1954), he pointed out, with characteristic perspicacity (ibid, p. 305, footnote 1; italics added):

“The general principles of stabilization have been illustrated here with particular reference to aggregate production in a closed economy, but they are of general applicability. They could equally well be used, for example, in investigating the stability of adjustments in international trade, or the problems involved in commodity price stabilization schemes.”

Thus, using the nonlinear multiplier-accelerator Keynesian model developed in Zambelli (2010), we get, for the aggregate dynamics of production, Y:

\[ \varepsilon \theta \dddot{Y}(t) = - (\varepsilon + (1 - c) \theta) \ddot{Y}(t) - (1 - c) Y(t) + C_0 + \varphi (v Y(t) - K(t)) \]  

\( (3) \)

The state space representation of which is given by the following three first-order, nonlinear, ordinary differential equations:

\[ \ddot{Y}(t) = Z(t) \]  

\( (4) \)

\[ \dot{Z}(t) = b \left[ C_0 - (1 - c) Y(t) - a Z(t) + \varphi (v Y(t) - K(t)) \right] \]  

\( (5) \)

\[ \dot{K}(t) = \varphi (v Y(t) - K(t)) \]  

\( (6) \)

\[ b = \frac{1}{\varepsilon \theta}; \quad a = \varepsilon + \theta (1 - c) \]

Where:

\( K(t) \): aggregate capital at time \( t \);
\( \varepsilon \): Robertsonian lag between the decision to consume and its realization;
\( \theta + \varepsilon \): Lundbergian lag between the time at which decisions are made to invest and its realization;
\( \theta \): half the construction time required for new equipment;

Relation (3) is the most direct extension of the linear multiplier-accelerator model in § 8 of the mathematical appendix in Phillips (1954, p. 321, ff.)
The next step, of course, is to link two trading economies with each other and consider the complete system of six nonlinear ordinary differential equations, in state space and block diagram representations. One way of doing this is given in Zambelli (op.cit), but it is not the only way.

Finally, two caveats to the above discussion on extensions are apposite, here:

First of all, it is not necessary, as Phillips and Goodwin seemed to have thought, to connect two Phillips Machines – or two analogue computers – to study the dynamics of coupled economies. One system of six coupled, nonlinear, ordinary differential equations, can be explored for possible attractors by means of a series of structured simulations, using one Phillips Machine.

Secondly, it is provably impossible to find closed form, analytical solutions to such a system of six coupled nonlinear ordinary differential equations: structured simulations are the only computational means at our disposal – for now.

§4. Brief Concluding Notes

“So you can imagine my pleasure when I arrived in Cambridge to find the Phillips machine, which could simulate arbitrary nonlinear dynamics. I found it a difficult machine and had troubles but, with occasional help from Phillips, I managed to keep it functioning for some years. …

[PS: I was very pleased that Phillips had 2 machines in London and I was able to show him (which he had doubted) that we could produce aperiodic, ‘chaotic’, motion10 with the two interconnected.]

Goodwin, 1991 [Letter to Nick Stern]

The decades of what we call policy nihilism span the thirty years from the Lucas critique (Lucas, 1977) to the eve of the Great Contraction – i.e., 1977 to 2007. The hallmark of these thirty years of monumentally ahistorical scholarship, particularly in macroeconomic theory and its policy bases, was, surely, a

10 It is just as well that the maestro, Richard Goodwin, places chaotic within quotes – since what he generated was what is now called the quasi-periodic paradox!
comprehensive ignorance of the pioneering works by the Phillips contributions prior to the ‘sanctification’, in vulgar neoclassical economics, of the Phillips Curve\textsuperscript{11} contribution in Phillips (1958).

The whole sorry tale of the Taylor Rule (Taylor, 1993), and its dominance in monetary policy discussions, is a further testimony to the errors of using intrinsically non-dynamic frameworks – i.e., non-endogenous dynamics – to discuss the pseudo dichotomy between rules and discretion. It is, at least for us, an astonishing historical fact that Phillips, already in his pioneering classic of sixty years ago, was naturally considering the issue of \textit{good}\textsuperscript{12} (cf. Taylor, \textit{op.cit.}, p. 195), not \textit{optimal}, policy rules. For, after all, how does one compute – or even show the existence of – an optimal policy rule in a nonlinear dynamical system, constrained by multiple objectives?

The point we wish to make is quite simple: Phillips (1954), together with the analogue device he constructed, and described in Phillips (1950), are the most fertile spring boards for a revival of a development of an applicable theory of economic policy. In them one finds the seeds of a re-orientation of an approach to simulational explorations to learn from structured experiments – just as he did with his wise and enlightened study of alternative policy regimes, within an explicitly developed endogenous macrodynamic model

That the model has been discarded is an indictment of the professions mono-maniacal devotion to \textit{ad hoc shockeries}\textsuperscript{13}. That the method of simulation was superseded by trivial and inappropriate econometrics is a further testimony to the methodological madness of the profession, at one of the frontiers of policy nihilism.

Above all, Phillips always knew the wisdom of remembering that a \textit{continuous} time dynamic model requires an analogue computing device to make sense of structured simulations.

\textsuperscript{11} Lipsey (1978) suggests (\textit{ibid}, p. 51) that the phrase was coined by Paul Samuelson in the sixth edition of his celebrated undergraduate text, Economics. Thus, we have been saddled with two regrettable phrases by the last of the Universal Economists – one in the 3\textsuperscript{rd} edition, \textit{the neoclassical synthesis}, and the other in the 6\textsuperscript{th} edition, the Phillips Curve!

\textsuperscript{12} In the true sense of Voltaire: \textit{Le mieux est l'ennemi du bien}.

\textsuperscript{13} Richard Day’s felicitous term to describe Lucasian Macroeconomics (Day, 1992, p. 180)!
REFERENCES


