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DISCUSSION PAPER SERIES

3 – 2013/II

"CARRY – ON – ACTIVITY" AND PROCESS INNOVATION

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AUGUST 2013

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Abstract

Economic growth driven by the creation of new ideas, knowledge, and innovations is an interesting and challenging phenomenon to be modelled and analysed. Dominant approaches, like Romer's endogenous growth models, Aghion-Howitt creative destruction growth models, RBC models, or the not so dominant ones, like that of the Neo-Austrians, of industrial structuralisms, or Nelson and Winter's evolutionary theory, emphasize the need for modelling innovations in a comprehensive manner but fail to encapsulate the intrinsic indeterminacies of the innovational process in an insightful manner. Romer (1986, 1990, 1993), in his seminal works, has stressed the importance of modelling ideas and knowledge and showed how ideas can be encoded as information bit-strings, using the chemistry set metaphor. Zambelli (2004, 2005) developed the idea further, modelling an innovation process as a Turing Machine (TM) process and applied the TM metaphor within Romer's endogenous growth model in an interesting way. But due to the time-less property of the production function, the dynamic interactions between the production and innovational processes could not be explored in detail. In this paper, we harness the intrinsic indeterminacies within the TM metaphor to model process innovations and analyse its dynamic interactions with the production processes within a time to build framework.

1 Introduction

In this paper we study a theoretical model where R&D is the search for a better production process. In order to make the concept of process innovation precise we adopt a particular interpretation of the Church-Turing thesis to claim that anything which is computed, that is: anything which is ‘produced’, can be in principle computed-produced by a Universal Turing Machine or, equivalently, by a Universal Constructor (see Chaitin, 2010: 73). Following the work of Velupillai (2000, 2010) and Zambelli (2004, 2005) we assume that the output can be encoded as a string, which is the input, the specific Turing Machine and the output to be fed to the Universal Turing Machine. The working of the Turing Machine is interpreted as being equivalent to the carry-on to completion of the product which does require the employment of labour. That is, a process innovation is defined precisely as the generation of the same output-string with a different Turing Machine. Between the different production processes, the most efficient is the one that is able to produce the output with a lower algorithmic complexity (see Zambelli, 2004, pp.165-7 and 2005, pp.238-46). Lower algorithmic complexity means lower necessity of labour to complete production. Whether a more efficient process is going to be implemented by the firms depends on its efficiency, i.e., its algorithmic complexity, but also on the market conditions and R&D policies. In this paper an endogenous model of process innovation is presented and studied.

The production of a commodity requires the use of means of production and time. Time to build framework plays a vital role in modelling macrodynamics and has been at the core of the Austrian and Neo-Austrian traditions (see Hicks, 1973; Amendola and Gaffard, 1998 and 2006), but also in other traditions (see Frisch, 1933, Kalecki 1935). The point that is central to these approaches is that between the moment in which a decision on production is taken and the moment in which the product is brought to completion, market conditions can change so as to modify the economic convenience of production. It is not only time that elapses, but resources (mostly labour) have to be used and financed at the same time as the production takes place. This asynchrony between the carry-on production and the delivery of the output for market sale may be fundamental in generating (explaining) business cycles and/or in shaping the traverse.

Both Frisch (1933) and Kalecki (1935) have captured this asynchrony between the work efforts and investments in what they have termed the carry-on-activity. That is: a function of time describing the activity necessary at each point of time for the product to be brought closer to completion. Practically, this carry-on-activity does represent the production process. Carry-on-activity is known also as manufacturing process management.

It is interesting to point out that in the seminal works of Tinbergen (1931), Frisch (1933), Kalecki (1935) and Goodwin (1951) the effort to complete a unit of output is constant all through the period of production and the problem of the different shapes of the carry-on-activity is not investigated. Likewise, Amendola and Gaffard (1998, 2006), which we can take as an example of the Neo-Austrian tradition (Hicks, 1973), do the same and the work efforts necessary to complete a unit of output are constant through time. The fact that market condition may cause the interruption of the completion of the product is obviously a different matter with respect to varying working efforts.

Kydland and Prescott (1982) in their famous `time to build' model have a production function, which is point input - point output. *Practically* it does not take time to produce and hence they have no carry-on-activity function: production occurs inside the period and/or *at most* there is one unit of time lag between in the moment in which production takes place and the output is completed. Romer (1986, 1990, 1993), in his work on the endogenous creation of knowledge, has emphasized the importance of ideas and knowledge and has focussed on its creation, but has not considered the fact that it takes time and effort to produce. Innovations do not involve the production process. In his Chemistry set metaphor [3], Romer discusses the *curse of dimensionality*[4], but leaves out discoveries and new ideas that innovate with respect to the production process.

Even Velupillai (2000: Ch. 9, 2010), and Zambelli (2004, 2005) who have expanded the implications of Romer's (1993: 68) suggestion to characterize an idea as being equivalent to a bit string by considering the generating process of a Turing Machine do not focus on the *time and effort* necessary to bring to completion a unit of output (Production) or on the time and efforts to generate a new productive idea (R&D).

In the following, expanding from the above mentioned works of Velupillai and Zambelli, we will present a model in which process innovations and the processes

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[3] As Romer (1993: 68) explains,

> [W]ithin the metaphor of the chemistry set, it is obvious what one means by an idea. Any mixture can be recorded as a bit string, an ordered sequence of 0s and 1s of length 100. The bit at position j is set to one if element j is included in the mixture… an idea is the increment of information that comes from sorting some of the bit strings into two broad categories: useful ones and useless ones. To represent this information we can add two more bits on the end of each bit string describing a mixture. These are set 00 if we know nothing about its properties, 10 if it is a useful mixture, and 01 if it is useless.

[4] For example, the number of different combinations of the chemistry set is 2^{100-1} (or approximately $6.3*10^{29}$) and another such example is that a shirt is made in 52 steps, so there are 52! ways of making a shirt, and since it is a large set, and not every method has been tried out so there is always some scope for improvement (Romer, 1993).

themselves are endogenously generated as Turing Machine equivalents in which it "takes time to compute", hence that it takes time to produce.

The importance of the Turing Machine as a mechanism encoding ideas and innovations is due to an adherence to a particular view of the Church-Turing thesis that allows us to consider productive discoveries as being equivalent to the strings generated as the result of *halting* Turing Machines. One of the fathers of Algorithmic Information Theory, Greg Chaitin (1995: x) has claimed that a "universal Turing Machine is, from a physicist's point of view, just a physical system with such a rich repertoire of possible behaviours that it can simulate any other physical system." If we consider the production of goods and services equivalent to the transformation of ideas into physical processes aimed at the production of commodities the importance of the Turing Machine metaphor should be evident.

Hence, by modelling the Research and Development activities as a concurrent search of new Turing Machines and as Turing Machines, we are bound to deal with the *halting* problem of the TMs and we are able to capture the intrinsic uncertainty of the R&D innovation processes (Zambelli, 2004; 2005). In so doing we avoid stochastic *ad-hockeries*. The key, then, is to conceptualise the dynamics of production and innovation process as a dynamically interacting phenomena and study its evolution. As we will see the *production process depends on the degree and magnitude of the innovation process, but also the innovation process depends on the production process*.

The conceptualization of production as a computational process forces us to view the system as being intrinsically dynamical and to model the evolutionary process in an insightful way. In this context, concepts such as viability and that of the traverse (see Dharmaraj, 2011), which are essential in distinguishing between the innovation and its economic feasibility, are given a rigorous content. The Neo-Austrian approach is a very useful framework that enables us to understand the dynamic behaviour of the economy but is limited in explaining the generation of new ideas, knowledge, and innovation. In this paper, we focus on the aspects of knowledge creation, which is *process innovation* and *labour saving mechanization*. In the characterization of a process, we will follow the Romer-Velupillai-Zambelli approach mentioned above, by considering a product to be produced with a process which is a bit string. Process innovations will be equivalent to *robotisation* (*automation*) of production.

The conceptualization of innovational processes as a process in time, using the Turing Machine metaphor (Zambelli 2004, 2005), enables us to model the evolution of process innovation in an insightful and non-stochastic way. Moreover, this endogenous model of process innovation will provide us with a valuable tool to study the concurrent effects of reducing the labour necessary to produce and the time to build. Therefore, the model of process innovation is then infused into a synthesis of traditional macroeconomic models where *'time-to-build'* plays a central role.

2 The Model: Time-to-Build, carry-on-activity and process innovation

> In algorithmic information theory, you measure the complexity of something by counting the number of bits in the smallest program for calculating it:
>
> Program → *Universal Computer* → output
>
> If the output of a program could be a physical or a biological system, then this complexity measure would give us a way to measure the difficulty of explaining how to construct or grow something, in other words, measure either traditional smokestack or newer green technological complexity:
>
> Software → *Universal Constructor* → physical system
>
> DNA → *Development* → biological system
>
> Chaitin (2010: 73)

Chaitin (2010) emphasizes that ideas, knowledge, innovations, technologies, and, even, economies can be considered as bits of information, which would enable the human/machine to produce/compute.

In this paper we assume that the product is encoded as a long string of 0s and 1s. For example, the following string of *n-digits*

01110010100010010100100011110010...1010010001100001010101010111 1101 can be seen as the encoding of a product. Such a string may be produced by several TMs. Note that there exists at least one TM that is able to produce the above string. This TM is the trivial TM that, operating on a blank tape, does print the string in a sequential manner. How to construct this TM is discussed in Zambelli (2005). If we define algorithmic complexity in terms of the quadruples (i.e., states) of the TM we can consider as the worst case for the production of the above *n-digits* string that in which the TM to produce it, has algorithmic complexity precisely equal to *n*. When a TM with lower algorithmic complexity with respect to the above worst case is found, by construction, we reduce the known complexity of the string. We can say that we have a better way to produce the string. Here, we make a direct analogy with the production process. We link the algorithmic complexity of the TM generating the *n-digits* string with the labour efforts necessary to conduct the computation. A process innovation is going to be defined as *robotization* of production and hence as a reduction (saving) of labour efforts.

The analogy with computation by TMs and production is straightforward. Labour is necessary to compute-produce the output and is necessary to operate the machines that allow a reduction of the working efforts through the use of a TM with lower complexity. Once the above is granted a crucial problem is that of modelling the discovery process. How can the new *labour effort reducing machines* can be discovered? The link with R&D is straightforward. R&D activity is defined as labour devoted to the scope of finding new TMs with lower complexity with respect to the known ones.

The R&D unit carries out research in order to find if there is any shorter algorithm than that of the object itself. As in the so-called 'real world', the R&D unit has a task and characteristics that are very different from the production unit. The production unit does not face uncertainty in the sense that the task of production is simply that of implementing well-known processes whose outcome is well defined. Using the TM metaphor we can say that the *n-digits* string can be produced with *certainty*, using an already halted TM.

On the other hand, the work of the R&D unit is subjected to high degrees of uncertainty. The outcome of trying out of new TMs and discovering whether that TM allows *robotization* of the process is highly uncertain. This is due to the intrinsic uncertainty related with the structure of the generated string, the innovation, that it is likely to be different from the encoding of the output or it is due to the existence of the halting problem, i.e., to the fact that the outcome of the tried out specific TM is unknown because it is not known whether the TM will halt or not.

2.1 The search of process innovations through investment R&D: the search process

The labour employed by a firm at any given time depends upon the firm's strategy - to carry out production with the given, already available production process and/or whether the firm is willing to search for better production processes, i.e. a better carry-on-activity. If the firm decides to invest in R&D, it will have to allocate resources, i.e. labour, for the search of this new process.

Without entering into the details of the actual search for new TMs here, let us just say that the search process is equivalent to *data compression*, which means a reduction of the number of states necessary for the production of the string or subsets of it.

We assume that the production of the output q

- is encoded by a digital string;
- that it has time to build equal to ε periods
- that for each period labour efforts are required to proceed in the computation;
- the efforts per unit of time necessary to bring production to completion depends on the available carry-on-activity;
- a process innovation is an improvement on the carry-on-activity and depends on whether a new TM with a lower algorithmic complexity has been found;
- R&D is the search for these new TMs – an increase in the number of researchers determines an increase in the number of TMs tried out.

In a way, the above procedure is in someways equivalent to the scheduling of work tasks necessary in order to complete an output. Below a scheduling of actual production with the associated work effort is reproduced.

Innovations are reductions of the labour efforts that are made possible; thanks to the discovery of a lower state TMs. Process innovations are the generation of new charts.

2.2 Production Process and Process Innovation

Production of goods, or ideas or knowledge or innovation, is essentially a transformation of a set of inputs, according to a set of rules (i.e., *algorithm*), to produce the desired output. The crucial difference between the production and innovation process is that the production processes are *deterministic* in nature while the innovational processes are *not*. That is, for the production processes that are being carried out with the inputs and the set of rules, we know *a priori* that it will produce the desired output. In the case of the innovational process, it is impossible to know, *a priori*, if the process that is being carried out will halt by producing an innovation or not. When we conceptualize production as a process, in time, then the time to build characteristics of the production take a central stage amongst all the factors that affect it.

Preliminary assumptions:

- Events occur in discrete time;
- There are a finite number of firms *(n)* that produce an identical output *(q)*;
- The output is perishable, i.e. once produced it cannot be stored;
- The output is consumed by the workers;
- During each time period the total output, *Q(t),* is sold at a uniform price, *p(t).*
- Workers do not save and use all their income to buy the output;
- Producers buy labour at a given wage, *w(t).* Wages are paid before the labour is delivered – i.e. the firms either have previously accumulated funds or borrow funds (either from the workers themselves or from a bank);
- The completion of a production, according to the production *blueprint*, requires time - i.e. it requires several periods (ε_i) to be completed - and it requires work effort which vary across periods (carry-on activity);
- The whole production knowledge and output description is encoded in terms of Turing Machines. Innovations and/or discoveries of new processes are in terms of the discoveries of halting TMs (as in Velupillai, 2000: Ch. 9, 2010: Ch, 10, and Zambelli, 2004, 2005).
- Each firm can decide how much to devote to the production of the final output or to R&D. The discovery by one firm is patented and cannot be used by other firms.
- There is an authority that decides:

- Interest rate and financial capital taxation rate. Hence (see below), the authority determines a net interest rate *r(t)* which can be also negative because it is the net between interest payments and taxation payments;
- Bankruptcy rules.

Firms decide production at time *t,* which will be completed at time $t+\varepsilon_i$. ε_i is the individual firm's *period of production,* which could be different with respect to the different firms and will depend on the specific research efforts and market condition. In this respect we follow the early mathematical formulation of Frisch (1933) and Kalecki (1935). Decisions of production (orders[5]) made at time $t-\varepsilon_i$ are planned to be delivered at time *t.*

$$Q^P(t)=\sum_{i=1}^{n} q_i^P(t)\ \textit{Aggregated actual real planned output (planned deliveries)} \qquad (1)$$

$$O^P(t)=\sum_{i=1}^{n} o_i^P(t-\varepsilon_i)\quad \textit{Aggregated real planned production (orders)} \qquad (2)$$

Clearly we have that the firms planned output, $o_i^P(t-\varepsilon_i)$, will become actual output $q_i(t)$ only in the case in which the plans are fulfilled. Planned production may not become actual output simply because there may be a lack of financing or because the prospects have changed and what was expected to generate *gains* at time *t* is suddenly expected to generate *losses* or because between the moment $t-\varepsilon_i$ and *t* there has been a process innovation that makes the adoption of a new process more convenient. Hence, the actual production realized or the deliveries (1a) will be

$$Q(t)=\sum_{i=1}^{n} q_i(t)\ \textit{Aggregated actual real output (actual deliveries)} \qquad (1a)$$

During each period the financial wealth of a firm will be given by:

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[5] In the context of this paper production decisions and 'orders' are equivalent, as originally used by Kalecki (1935).

$$FW_i(t) = \mathrm{Rev}_i(t) - Exp_i(t) + FW_i(t-1) \tag{3}$$

Clearly there will be firms that will have a positive financial net worth (credit) and others that will have a negative financial net worth (debt). $\mathrm{Rev}_i(t)$ are the revenues of firm *i* at time *t* and $Exp_i(t)$ are the expenditures at time *t*.

The revenues at time *t* are the revenues from sales of produced goods and the revenues due to interest payments from others

$$\mathrm{Rev_i}(t) = \mathrm{Rev_i^q}(t) + \mathrm{Rev_i^F}(t) \tag{4}$$

While the expenditure at time t is given by payment of wages and the payment of interests to others

$$Exp_i(t) = Exp_i^q(t) + Exp_i^F(t) \tag{5}$$

Assuming the same interest rate for assets and liabilities, we have the following 'financial cost':

$$\mathrm{FC_i(t)} = \mathrm{Rev_i^F}(t) - Exp_i^F(t) = r(t)FW_i(t-1) \tag{6}$$

Clearly whether $\mathrm{FC_i(t)}$ would be positive or negative depends on whether $r(t)$ is positive or negative and/or on whether $FW_i(t-1)$ is positive or negative.

The revenues from sales of the produced output is given by

$$\mathrm{Rev_i^q}(t) = p(t)q_i(t) \tag{7}$$

In this model, the expenditure for production purposes is an expenditure on the only factor of production, which in this model is labour:

$$Exp_i^q(t) = w(t)L_i^{tot}(t) = w(t)\left(L_i^q(t) + L_i^{R\&D}(t)\right) \tag{8}$$

The output q_i can be produced by several alternative techniques, which all include different working efforts. In essence, there are different processes that imply different work efforts distributed in different sequences in between periods. A technique can be described by an array of time indexed labour inputs, the *carry-on-activity*, that are necessary in order to bring a project to completion, $\ell^{Z_i} = \lfloor \ell(0), \ell(1), \ell(2), \ldots, \ell(\varepsilon^{Z_i}) \rfloor$. Each firm has a production possibility sequence, which is described by: $(\ell^{Z_i}, \varepsilon^{Z_i})$. This would be the firm production process as long as a process innovation will not take place. Therefore, a decision of production made at time $t - \varepsilon_i^{Z_i}$, to produce a unit of the quantity q_i^t would require a proportional $\omega_i^{t-\varepsilon_i^{Z_i}}$ use of labour

$$\ell(t - \varepsilon_i^{Z_i}), \quad \ell(t - \varepsilon_i^{Z_i} + 1), \quad \ell(t - \varepsilon_i^{Z_i} + 2), \quad \ldots, \quad \ell(t) \rightarrow \quad 1 \tag{9}$$

$$\omega_i^{t-\varepsilon_i^{Z_i}} \ell(t - \varepsilon_i^{Z_i}), \quad \omega_i^{t-\varepsilon_i^{Z}} \ell(t - \varepsilon_i^{Z_i} + 1), \quad \omega_i^{t-\varepsilon_i^{Z_i}} \ell(t - \varepsilon_i^{Z_i} + 2), \quad \ldots, \quad \omega_i^{t-\varepsilon_i^{Z_i}} \ell(t) \rightarrow q_i^t \tag{10}$$

The focus in this paper is on *process innovation*. Process innovation can take the form of different work efforts per unit of time or of a reduction of the period of production, or both. In the sequel, we will assume that the period of production is fixed. Hence a discovery is the discovery of new, halting Turing Machines that implies a reduction in the work efforts per unit of time.

The discovery can take place if a firm invests in R&D. During each period, the firm *i* makes a decision of production $o_i(t)$ which implies a completion at $t + \varepsilon_i^z$. At every given point of time, the firm decides what percentage of its labour force is to be employed for the new and on-going projects and for conducting new R&D activities. The total labour demand of firm *i* at time *(t)* is given by

$$L_i^{tot}(t) = L_i^q(t) + L_i^{R\&D}(t) = \sum_{j=0}^{\varepsilon_i^Z} \omega_i^z (t - \varepsilon_i^Z + j) \ell_i^Z(j) + L_i^{R\&D}(t) \tag{11}$$

The *total* labour demand will be given by:

$$L^D(t) = \sum_{i=1}^{n} L_i^{tot}(t) \tag{12}$$

Assuming that the labour supply is inelastic to wages we have: $L^S(t) = L^D(t)$.

The labour employed by the firms at every given point of time depends on the revenue obtained from the sale of output commodities. If the revenue generated is less than the cost incurred (i.e. amount of wages paid for the labour), then the financial imbalance will reflect on the new projects, and in some cases even the on-going projects may have to be truncated due to shortage of credit or external financial resources.

The total output will be given by:

$$Q(t) = \sum_{i=1}^{n} q_i(t) \tag{13}$$

The output $Q(t)$ is not necessarily equal to what is planned, $Q^P(t)$, as in equation (1). The reason is that during production the firm may interrupt the production, or a process innovation may change the time to delivery of the original decision. Given our assumptions we have the following market clearing price determination:

$$p(t) = \frac{w(t)L^D(t)}{Q(t)} \tag{14}$$

Here we also assume that the process innovation can be patented and hence the innovation is associated to an individual firm. Obviously for a given level of employment the firm is confronted with a trade-off between deciding to use the labour force in the production of the final output or in R&D activity. Both variables $L^D(t)$ and $Q(t)$ are the result of past decisions made by the *n* different firms and hence they all depend on the level of technological progress, on the individual firm's access to technology. The natural and unavoidable asynchronicity between the moment in which a decision of production is taken and the moment in which the product is completed (i.e. it is delivered to the market) makes it impossible for the firms to know whether their decisions will determine a positive or a negative cash flow – the whole will depend on the overall discoveries occurring during the period, the financial and taxation conditions, the employment levels and the taxation rates, and so on. Furthermore, the model closure will require a specification of the firm level decisions of production, the allocation of resources to R&D. Once the closure has been provided the dynamics of the model can be studied through repeated simulations. Simulations will be parametric. This means that by changing some values of the parameters, different dynamic evolution will be generated. Once the simulation is started our approach does not require any stochastic element to be introduced, ever.

First we will study the dynamics of the model through several simulation runs and with policy variables unchanged.

Second we will run the same simulations as above, but with changed policy variables. That is with different policy rules for:

- the interest rate and financial capital taxation rate, *r(t);*
- bankruptcy rules.

Furthermore, it needs to be noted that in the context of this paper, credit is created exogenously. We have that $\sum_{i=1}^{n} FW_i(t) \equiv 0$ and that $\sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{FC_i(t)} = 0$.

The total stock of credit money is then given by:

$$M(t) = \sum_{i=1}^{n} \begin{cases} FW_i(t) & \text{if} \quad FW_i(t) > 0 \\ 0 & \text{if} \quad FW_i(t) \leq 0 \end{cases} \tag{15}$$

Clearly, the policy variable *r(t)* (which is both an interest rate and a capital tax rate) can influence the distribution of financial wealth and will have an effect on the dynamic evolution and on the innovation process (carry-on-activity and so on) and on the variables such as aggregate output and *M(t).*

3 Simulations

The above model can be studied through simulations. Here we are assuming that the common feature of the n firms is that they produce the same output, but the technological innovations and the organizational structure of the firms are different and their development and economic survival depends on the activity of the R&D department. Clearly the firm has to, on one hand, complete production, and on the other hand, invest in R&D. Investment in R&D is an investment in the future and the revenues from this activity are highly uncertain because they will depend not only on whether the R&D activity is successful, but also on what the market conditions and the economic state of the firm will be in the future, i.e. at the moment in which the discovery will be made. The market conditions would depend on the different strategies of the n firms and on the discoveries. Firms with low investment in R&D will have, in the short run, better market performance, but this does not imply that the same firm will not have high R&D performance in the future. Clearly, the higher the revenues from sales of the output the higher would be the *capacity* to invest in R&D in absolute terms. On the contrary, an aggressive firm investing most of its resources in R&D might end up doing very little R&D, simply because of the lack of revenues from the sales of the physical output.

Having the above example in mind, one can speculate on whether it would be advisable to subsidise the activity of the more aggressive firm, which would almost certainly suffer in the short run. In the context of our model, a subsidy requires financing through taxation. In order to focus on some important features, we will assume that the nominal wage rate is fixed and that employment is also fixed. Labour income is also *not* taxed. What we have is that a subsidy does in fact imply a redistribution of purchasing power among the firms. The transfer from profit making firms, which accumulate financial wealth, can be made with a negative interest rate (that is the profit-capital tax rate is above the monetary interest rate) and assuming that at first the losses will be made by the firms that invest in R&D, this mechanism should allow for the detection of new techniques or better organizational structures. On the contrary, a reinforcement of the positive economic performance can be obtained with a positive interest rate (that is when the monetary interest rate is above the profit-capital tax rate). Figure 1 reports three different aggregate evolutions of the output, whose difference is to be found in the three different exogenous tax-interest rates and *ceteris paribus*.

In our model, an interest rate value different from zero implies a redistribution of financial wealth between the different firms. Given that the financial wealth is not accumulated, but is used to employ labour, different levels of financial wealth would imply different capacities to employ labour and hence different amounts of labour force to be used in the R&D department. The *n* firms are ordered in terms of their propensities to invest in R&D and it is not said that the firms having high relative investments in the R&D departments will be the ones most successful. The difference in evolution reported in the Figures is to be explained by the fact that the firm that may have implemented a very important innovation goes bankrupt before being allowed to innovate.

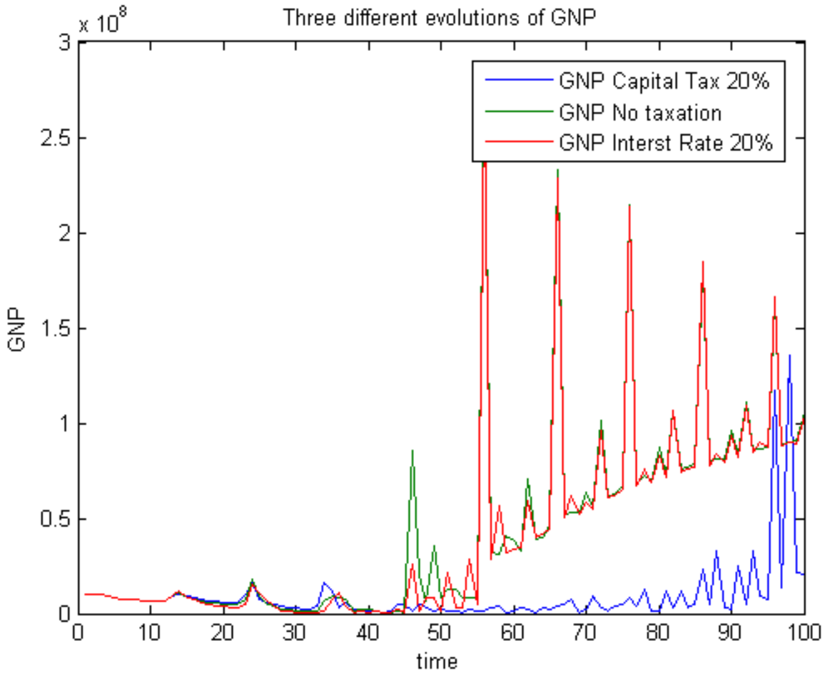

Figure 1

Another interesting feature of our experiments is to be found in the unavoidable short-run fall of output, which is a consequence of allocating resources from physical production to production of knowledge (R&D). But when the firms' R&D discovers new information bit strings that reduce the algorithmic complexity of the production blueprint by several orders of magnitude, we observe that the output of the firms increases drastically. As the innovative firm cannot save a portion of its profit for the next time period, the excess money is then reinvested in new projects and R&D activities. Therefore, when that new project, with innovative *carry-on* activity function, comes to completion there will be a significant increase in the output, but as the economic system as a whole adapts over time and the output is smoothened out until a new discovery or innovation is made. This behaviour captures the disruptive nature of innovation and its effects on production, labour and money.

It is interesting to note that when the economy decides to employ a portion of the production labour for R&D activities, the GNP decreases thus increasing the prices of commodities. The firms that do not carry out R&D make profit and, with the profits, the firm employs more labour for production. The scenario changes only when the R&D firms start to innovate, thus reducing the cost of production, while the cost of production of the firm with no R&D remains the same.

Figure 3 reports a typical dynamic evolution that follows the decision to invest in R&D. Without R&D the output would have continued as a straight line, but the divergence of resources towards R&D does decrease production and hence total sales. It is only after a transient period that, due to the increase in productivity, we observe an increase in total output (See figure 3).

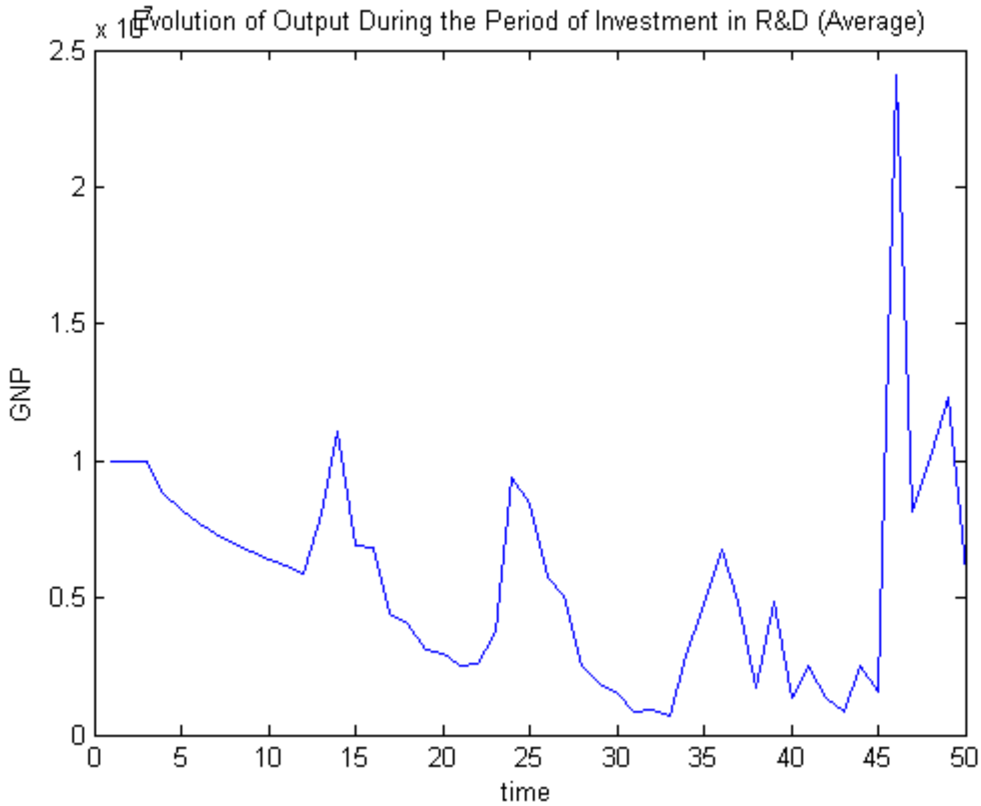

Figure 2

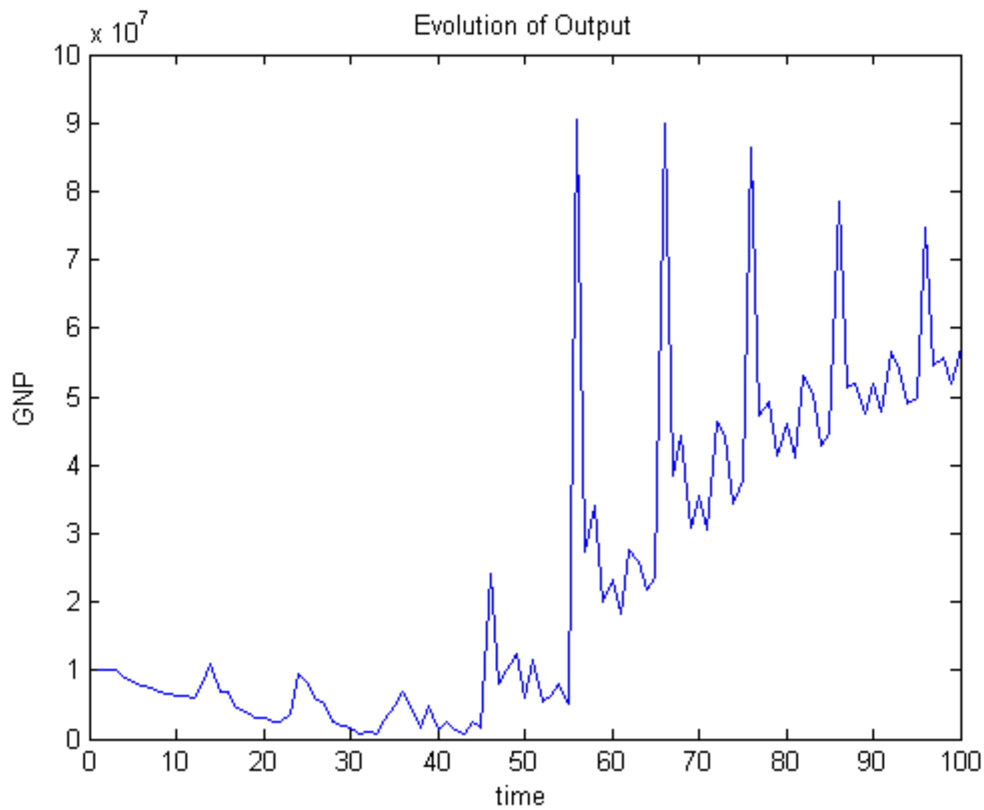

Figure 3

In our model the knowledge lost when bankruptcy occurs cannot be transferred as a bonus to the other firms, it is simply lost. Another leakage is determined when the firm is forced to cut down or truncate a previously started production plan. The work effort put into past projects, when not brought to completion, is lost. The competition among the different firms, the drive to find better processes and the specific competition of our model (captured by market clearing conditions) leads in almost all cases to the survival of just one firm that would end up employing the whole labour. Given that we have only one output that is produced and that the firms and workers have access to the same market, this result is not surprising at all.

Figures 1 to 3 show different evolutions in terms of different interest rates.

In Figure 4, we report a graph where statistics concerning different performances of aggregate output are collected as a function of the interest rate r(t) (from – 20% to + 20%). The graph reports the average value of the output for 100 different runs where the only difference is the TMs space. As seen from the graph, we can conclude in the context of our model that the policy of transferring resources from profit making firms to loss making firms with redistribution through taxation (negative interest rate) is not rewarding. This is not an obvious result. We would have expected the contrary. Clearly during the period in which a firm invests in R&D the firm is likely to make lower revenues from sales and hence lower resources would be available for investment in R&D. Transferring resources towards these loss making firms is a policy that should allow faster discoveries. But our results indicate that this is not the case. Positive interest rates imply that the firms that are incurring losses have to borrow from the profit making firms. In the context of our model, it turns out that a trade-off between resources to be devoted to production of the final good and to R&D is advisable. What our results indicate is that the support to profit making firms allows for a sort of Schumpeterian mechanism in which the firms that do innovate first, but that do at the same time invest in production, are those that are most successful. In our case, this leads to an increase of the total output as well. As Figure 3 indicates, this type of mechanism implies a drop in production at first (what Aghion and Howitt would, maybe erroneously, call destruction) and subsequently an increase of the output. The result of Figure 4 indicates that after 100 periods the highest average production is to be associated with interest rates higher than 6%.

Another very interesting result can be derived from studying the number of times in which a firm has survived. We are working with 10 firms characterized by their R&D intensity going from 0 R&D to 45 % of the employed labour. Given the true and deep uncertainty of the innovation process captured by our characterization of knowledge in terms of the TM and the importance of the market conditions present at the moment in which a discovery is made, it would be interesting to have statistics concerning the

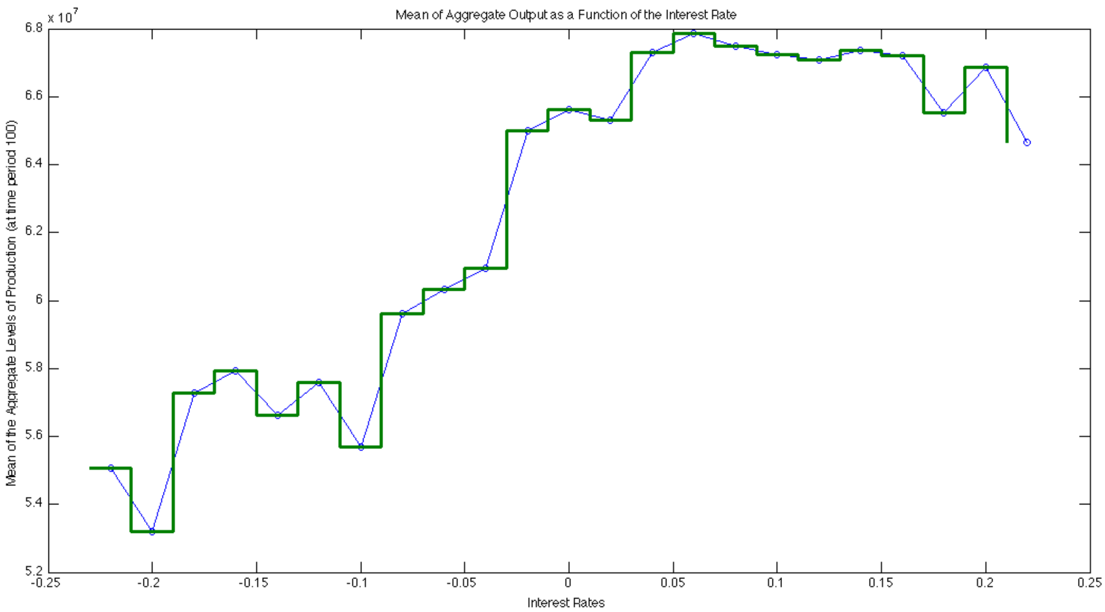

Figure 4

performance of the different firms, i.e a measure of the times in which the different firms would turn out to be successful. Or, equivalently, a statistic concerning the number of times when a firm would go bankrupt.
We have found that in our model the "strategy" which turns out to be the most successful is to invest around 15% of the available labour efforts into R&D.

It should be noted that with a positive rate of interest the firms that decide to carry out more R&D but fail to innovate goes bankrupt quickly, while the firm with no R&D activities survives for a longer period. In these situations, credit policy may play a vital role in determining the survival of the R&D firms. A negative rate of interest may reduce the aggregate output of the economy and therefore it would be interesting to further investigate the dynamics of the economy when more credit is introduced within the economic system, along with various other viability and bankruptcy mechanisms. The simulations show how an economy evolves in disequilibrium and it would be a very interesting to further enhance and investigate the model for different policy parameters.

4 Conclusions

Our scope has been that of investigating how to make the carry-on-activity function endogenous. What we have elaborated on is the observation that process innovation is nothing but a change (improvement) of the carry-on-activity function. The model of section 2.2 has very interesting characteristics. The fact that the distinction between the moment in which a discovery is made, the moment in which the investment decisions are made and the way in which these decisions of production are brought to completion have all relevance for the determination of the individual firms and of the market conditions, which in turn do influence the formation of the carry-on-activity function. Frisch (1933) and Kalecki (1935) did operate with constant and exogenously given carry-on-activity functions – and we have shown how to remove that limitation.

Our approach may have some relevance also for modern (real) business cycles, Kydland-Prescott-style, for endogenous growth models, Romer-style or for the creative-destruction models of the Aghion-Howitt type. In all these studies, the issue that it does take time to build and that production has to be seen as a process is not discussed in a meaningful matter. In all these models technological progress is just a simple and immediate change of the Cobb-Douglas production function (or its variants).

Clearly, what we have presented here is an embryonic model and several improvements can be made. In particular, it would be appropriate to relax the assumption of full employment of labour resources and it would beinteresting to consider a model with multiple products where innovations are to be seen not only as process innovations, but also as product innovations.

The whole exercise requires keeping in mind the richness of the TM's equivalent computations and discoveries.

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