Specialization patterns and reduction of CO$_2$ emissions

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Specialization Patterns and Reduction of CO2 Emissions.
An Empirical Investigation of Environmental Preservation and Economic Efficiency
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Abstract
In this paper we show that, when given a specific vector of net output of a group of countries, there are many specialization patterns that would allow its production and at the same time would allow substantial reductions of global CO2 emissions.
Empirical estimates of these potential reductions are provided using a set of 30 Input-Output tables during the period 1995–2009. We show that the environmental benefits may not be incompatible with improvements in terms of economic efficiency.

Keywords: CO2, environment, specialization patterns, net product possibility frontier, resources allocation, Input-Output

JEL classifications: E1, E23, F15, F42, P11

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Introduction

The reduction of carbon dioxide (CO2) emissions was set as one of the main goals of the Kyoto Conference of 1997. Specifically, the Kyoto Protocol, which entered into force in 2005, had the objective of reducing the global emissions of CO2 and of five more gases\(^1\) by 5.2% with respect to the 1990 emissions\(^2\). Similarly, reducing emissions is one of the main pillars of the strategy of the Paris Agreement, which entered into force in 2016 and aims to control climate change by limiting global warming.

The emissions of CO2 on a global scale are strictly linked to the available productive technologies in two different ways. On the one hand, technological progress makes available new methods of production that allow us to reduce energy consumption and pollution. On the other hand, as long as the technologies adopted by individual countries imply different CO2 emissions, the specialization patterns also affect the environment in different ways and intensities.

This second relation between technologies, specialization patterns, and CO2 emissions is the main topic of this paper. Extending an approach based on Input-Output (I-O) tables and subsystems that has been developed to study Comparative Advantages (CAs) (Boglioni and Zambelli, 2016; Boglioni, 2017), we show that a specific vector of net product can be produced through many different specialization patterns. This implies that once the vector of the final demand and the technologies are given, it is possible to identify the CO2-minimizing specialization pattern.

The methodology adopted here is similar to the one suggested by (Miller and Blair, 2009, p. 457) but the application of the notion of subsystems as in the Sraffa-Gossling-Pasinetti tradition is specific of our approach (Sraffa, 1960; Gossling, 1972; Pasinetti, 1980), as are the algorithms that we have built to perform the computation.

Using a set of I-O tables for 30 countries taken from the World I-O Database (Timmer et al., 2015), we compute the reduction of CO2 emissions for a case in which national productive specializations are allowed. We reach the con-\[1\]Methane, nitrous oxide, sulfur hexafluoride, hydrofluorocarbons and perfluorocarbons.
\[2\]The first enforcing period of the protocol ended in 2012 but in December 2012 the parties of the Kyoto Protocol adopted the Doha Amendment to the Kyoto Protocol, which basically revises some of the articles and renews the goals of the Protocol. The Doha Amendment sets new objectives for the participating countries, with a view to reduce emissions of the six gases that were in the original Kyoto protocol plus a new gas—nitrogen trifluoride—by 18% with respect to the 1990 levels during the period 2013–2020. The Doha amendment has not still entered into force.
clusion that improved coordination among countries may have a strong impact on the environment.

This result is somewhat compatible with the conclusions of Carbone et al. (2009). Their analysis is based on a different theoretical ground with respect to the one adopted in this paper—General Equilibrium Models and Game Theory—and they claim that international agreements on carbon emissions may be an effective way to reduce CO2 emissions. Our approach differs from that of Carbone et al. (2009) because the empirical results (Boglioni and Zambelli, 2016; Boglioni, 2017) show that coordination may, in general, be desirable because of market inefficiency.

Several studies concerned with reduction of CO2 emission have also addressed the trade-off between environmental regulation and economic efficiency, which is a sort of conventional wisdom in economics (Rauscher, 2003, p. 1410). A version of this “orthodox” view has been provided by Palmer et al. (1995), according to which environmental regulations would result in a distorted set of prices, which must imply a damage for firms’ profits and competitiveness, and hence a loss of wellbeing.

However, in this paper it is shown that these kinds of trade-offs would emerge on a global scale only in a case in which the theory of CAs worked well. A key assumption of this theory suggests that if markets were left free to work, then the “invisible hand” would naturally lead countries to specialize in those sectors in which they had a CA, in such a way that the overall net output would be maximized (Samuelson, 2001, p. 1205).

If this assumption of well functioning international trade markets and of efficient specializations could be proven to hold, then any kind of intervention that aimed to modify the specialization patterns to reduce CO2 emissions would result in a lower net output and, hence, in a loss of economic efficiency. But since international trade markets and national economies do not seem to be efficient, we show that there is scope for reducing CO2 emissions and improving the vector of net product at the same time.

Although this study is focused on CO2 emissions, this does not mean that the effect of other air pollutants is ignored. We will explain in this article that there is a specific focus on those specialization patterns which allow us to reduce CO2 emissions under the condition that the emissions of the other air pollutants available in the WIOD database cannot increase. The additional gases considered are: methane, nitrous dioxide, nitrogen oxides, sulfur oxide, carbon monoxide, non-methane volatile organic compounds,

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3The article by Palmer et al. (1995) came as a response to a series of studies summarized in Porter and van der Linde (1995), who argued that environmental regulations would result in an incentive to innovate that would fully offset the cost of complying with the regulation.
The rest of this paper is structured in five sections. In Section 1 the concept of subsystems, which is the fundamental tool of the minimization algorithm, is introduced, along with an explanation of how to use it to study specialization patterns. In Section 2, the concept of Net Product Possibility Frontiers is presented and it explains how it will be used to study CO2 minimizing specialization patterns. In Section 3 the dataset that we have used is briefly described and in Section 4 the empirical results are discussed. The conclusions are devoted to a summary of the results and they also offer some comments on their implications.

1 The environmental impact of different specialization patterns

To explain how different specialization patterns affect the environment, it is convenient to start by way of an example from a hypothetical economic system, as follows:

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input</th>
<th>Labor</th>
<th>Gross Output</th>
<th>Net Output</th>
<th>CO2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron</td>
<td>2</td>
<td>2/5</td>
<td>16</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Coal</td>
<td>5</td>
<td>2/5</td>
<td>14</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Wheat</td>
<td>6</td>
<td>1/5</td>
<td>9</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Tot</td>
<td>13</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>13</td>
</tr>
</tbody>
</table>

The three industries produce iron, coal, and wheat using the same three goods as inputs plus labor. To produce these goods, the transformation process of the inputs into the outputs generates emissions, for example in CO2, which can be treated as an additional output of the system.

We want to study how this system would look like if it specialized in some sectors. To do this, it is convenient to introduce some mathematical symbols to represent a generic I-O system, such as the one in Tab. 1. Where $A$ denotes the input matrix, $l$ denotes the labor vector, and $b$ denotes the gross output vector. It is useful to introduce the concept of social or national net product, which is that part of the national gross product that goes to the final demand, which is denoted with $y$. In matrix notation, the equation to compute the national net product is as follows:
\[ y = (\text{diag}(b) - A)'\mathbf{\iota} \]  \hfill (1.1)

where \( \text{diag}(b) \) is a diagonal matrix with the gross output vector \( b \) on its main diagonal and where \( \mathbf{\iota} \) is the summation vector; that is, all entries are equal to 1. In the case of the system above \( y' = [3, 5, 2] \).

As explained in Boglioni and Zambelli (2016) and Boglioni (2017), a subsystem is a fundamental concept in the study of specialization processes. The notion of a subsystem was introduced by Sraffa in the first appendix of his work *Production of Commodities by Means of Commodities* as a “smaller self-replacing system the net product of which consists of only one kind of commodity” Sraffa (1960, p. 105).

Basically, in a subsystem the means of production and the labor which is directly and indirectly necessary and barely sufficient to produce a specific net output in just one sector are considered. To compute a subsystem, each row of \( A \), as well as the relative amount of labor and gross output, must be reproportioned in such a way that each component of the net output vector is 0, except for the commodity in which we are interested. We use \( \bar{y}_i \) to denote the vector \((0, ..., y_i, ..., 0)'\), where \( i \) identifies the sector. To find a subsystem, we have to compute a reproportioning vector \( x_i \) such that

\[ (\text{diag}(b) - A)'x_i = \bar{y}_i \]  \hfill (1.2)

from which we have

\[ x_i = ((\text{diag}(b) - A)')^{-1}\bar{y}_i \]  \hfill (1.3)

Then, a subsystem is given by the triple

\[ A_i = \text{diag}(x_i)A \]
\[ l_i = \text{diag}(x_i)\mathbf{\iota} \]
\[ b_i = \text{diag}(x_i)b \]  \hfill (1.4)

where the index \( i \) is relative to the different industries, sectors, or produced commodities.

A fundamental property of subsystems is the “additive property”, where the sum of the subsystems gives back the original system; see Gossling (1972). Mathematically, if we have \( n \) sectors,
Assuming that the vector of CO2 emissions, denoted co2, are a linear function of the gross product vector b, we can compute the CO2 emission vector co2_i in the same way, that is to say

\[ co2_i = diag(x_i)co2 \]

\[ CO2_i = co2_i \times l \]

By adding up the elements of co2_i as in eq. 1.7, we can compute the total CO2 emission that is directly and indirectly implied in the production of the net product y_i in Sector i. In this paper, the symbols in italics denote a matrix, a vector, or a scalar relative to the subsystems.

The three subsystems with CO2 relative to Tab. 1 are reported in Tab. 2.

By applying the equations in 1.5 to the three subsystems in Tab. 2 we would obtain the values of the original system of Table 1; that is, 

\[ A = A_{iron} + A_{coal} + A_{wheat}, \]

\[ l = l_{iron} + l_{coal} + l_{wheat} \] and 

\[ b = b_{iron} + b_{coal} + b_{wheat}. \]

Please note that a superficial reading of Table 1 would indicate that the iron sector is responsible for 5 units of CO2 emissions, but when we read the data in terms of subsystems we can see that the production of a surplus in iron implies a much lower level of CO2 emissions (2.69), while the CO2 emission associated with the production of coal goes from 6 to 6.93 and that of wheat from 2 to 3.38. Although the total amount of emissions would be the same, the imputed values are different.

If we consider an autarkic situation, then a country has two possibilities to reduce CO2 emissions. The first is to reduce CO2 by introducing new methods of production and the second is to change the production of the net output in favour of products that imply lower CO2 emissions. In this paper we focus our attention on the second CO2 reducing factor. The primary non-producible resource of the system is represented by labor.

Using this information on the subsystems, we can analyse what the original system would look like if it specialized in the three sectors considered. To do this, it is sufficient to rescale all of the elements of a subsystem for the reciprocal of the quantity of labor involved in it—the total quantity of labor.
Table 2: The subsystems relative to the economic system in Tab. 1

<table>
<thead>
<tr>
<th>$A_{\text{iron}}$</th>
<th>$l_{\text{iron}}$</th>
<th>$b_{\text{iron}}$</th>
<th>$\bar{y}_{\text{iron}}$</th>
<th>$\text{co2}_{\text{iron}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.65 0.97 0.65</td>
<td>0.13</td>
<td>5.19</td>
<td>3</td>
<td>1.62</td>
</tr>
<tr>
<td>0.64 0.51 0.25</td>
<td>0.05</td>
<td>1.78</td>
<td>0</td>
<td>0.76</td>
</tr>
<tr>
<td>0.90 0.30 0.45</td>
<td>0.03</td>
<td>1.35</td>
<td>0</td>
<td>0.30</td>
</tr>
<tr>
<td>2.19 1.78 1.35</td>
<td>0.21</td>
<td>/</td>
<td>/</td>
<td>2.69</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$A_{\text{coal}}$</th>
<th>$l_{\text{coal}}$</th>
<th>$b_{\text{coal}}$</th>
<th>$\bar{y}_{\text{coal}}$</th>
<th>$\text{co2}_{\text{coal}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.81 1.22 0.81</td>
<td>0.16</td>
<td>6.48</td>
<td>0</td>
<td>2.03</td>
</tr>
<tr>
<td>3.47 2.78 1.39</td>
<td>0.28</td>
<td>9.73</td>
<td>5</td>
<td>4.17</td>
</tr>
<tr>
<td>2.20 0.73 1.10</td>
<td>0.07</td>
<td>3.30</td>
<td>0</td>
<td>0.73</td>
</tr>
<tr>
<td>6.48 4.73 3.30</td>
<td>0.51</td>
<td>/</td>
<td>/</td>
<td>6.93</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$A_{\text{wheat}}$</th>
<th>$l_{\text{wheat}}$</th>
<th>$b_{\text{wheat}}$</th>
<th>$\bar{y}_{\text{wheat}}$</th>
<th>$\text{co2}_{\text{wheat}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.54 0.81 0.54</td>
<td>0.11</td>
<td>4.32</td>
<td>0</td>
<td>1.35</td>
</tr>
<tr>
<td>0.89 0.71 0.36</td>
<td>0.07</td>
<td>2.49</td>
<td>0</td>
<td>1.07</td>
</tr>
<tr>
<td>2.90 0.97 1.45</td>
<td>0.10</td>
<td>4.34</td>
<td>2</td>
<td>0.97</td>
</tr>
<tr>
<td>4.32 2.49 2.34</td>
<td>0.28</td>
<td>/</td>
<td>/</td>
<td>3.38</td>
</tr>
</tbody>
</table>

has to be set equal to unity. For example, to simulate a complete specialization in iron, all of the elements of the first subsystem must be divided by 0.21.

To identify a system in which a full specialization has occurred, we use a hat above the symbols. For example, $\hat{A}_{\text{iron}}$ denotes the matrix of the physical inputs of a system in which the net product of iron has been maximized.

The specialized systems are reported in Tab. 3.

It can be noted that the total CO2 emissions in the three cases are different, and they would be minimized if the country fully specialized in wheat. That is, CO2 would be minimized if the consumption or surplus of the system would be only in wheat with the production of iron and coal limited only to the quantities necessary to produce wheat.

If we assume for simplicity that the actual produced net output is also the desired net output of the system, then here we face a trade-off: a decrease in CO2, which may be considered to increase social welfare, would be associated with a change in consumption, which may be considered to
Table 3: Specialized systems. Each of the following Tables shows what the economic system in Tab. 1 would look like under the hypothesis of complete specialization in one of the three sectors that compose the system in Tab. 1.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input</th>
<th>Labor</th>
<th>Gross Output</th>
<th>Net Output</th>
<th>CO2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron</td>
<td>2</td>
<td>2/5</td>
<td>16</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>Coal</td>
<td>5</td>
<td>2/5</td>
<td>14</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Wheat</td>
<td>6</td>
<td>1/5</td>
<td>9</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 4: The hypothetical economic system of Country 2
The CO2 produced in the first and the second sector is different, while, for the sake of the argument, the second country has the same means of production, the same gross output and, hence, the same net output as the first country.

Using the proper algorithm, such as the one described in Appendix A, we can study how they could be specialized in order to keep fixed the sum of their surplus and, at the same time, have globally lower emissions of CO2. The two countries’ vectors of surplus require that the overall surplus to be produced is 6 in iron, 10 in coal, and 4 in wheat. Clearly, the total labor employed must be 1 for each country. The CO2 emissions in the non-specialized, autarkic case amounts to 25.

The CO2-minimizing specialization patterns of the two countries are reported in Tab. 5.

**Table 5: CO2-minimizing specialization patterns.** The table shows how the economic systems in Tab. 1 and in Tab. 4 should have to specialize in order to minimize the global CO2 emissions while at the same time maintaining the sum of their original vectors of surplus.

<table>
<thead>
<tr>
<th>$\tilde{A}^1$</th>
<th>$\tilde{b}^1$</th>
<th>$\tilde{y}^1$</th>
<th>$\tilde{co}_2^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.42</td>
<td>3.63</td>
<td>2.42</td>
<td>0.48</td>
</tr>
<tr>
<td>3.23</td>
<td>2.59</td>
<td>1.29</td>
<td>0.26</td>
</tr>
<tr>
<td>7.71</td>
<td>2.57</td>
<td>3.86</td>
<td>0.26</td>
</tr>
<tr>
<td>13.37</td>
<td>8.79</td>
<td>7.57</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>12.50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\tilde{A}^2$</th>
<th>$\tilde{b}^2$</th>
<th>$\tilde{y}^2$</th>
<th>$\tilde{co}_2^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.58</td>
<td>2.37</td>
<td>1.58</td>
<td>0.32</td>
</tr>
<tr>
<td>6.77</td>
<td>5.41</td>
<td>2.71</td>
<td>0.54</td>
</tr>
<tr>
<td>4.29</td>
<td>1.43</td>
<td>2.14</td>
<td>0.14</td>
</tr>
<tr>
<td>12.63</td>
<td>9.21</td>
<td>6.43</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10.45</td>
</tr>
</tbody>
</table>

The symbol over the letters, such as $\tilde{A}^1$, identifies a specialization pattern in which the global CO2 has been minimized, while the superscript identifies the country.

The constraint on labor is satisfied, while summing up $\tilde{y}^1$ to $\tilde{y}^2$, we obtain a global surplus of 6 in iron, 10 in coal, and 4 in wheat, which is the original level. However, the total CO2 emission is now 22.95, which is 91.8% of the original.

A solution of this kind is viable as long as the two countries can share their net product in order to satisfy its original or autarkic domestic demand. Therefore, minimizing CO2 emission requires the existence of a
mechanism that redistributes the global net output. It is very likely that the decrease in CO2 emissions would be larger for those cases in which the methods of production of the different countries differ.

2 Environmental preservation and CAs

Suppose that there are many countries and that there is a coordination mechanism that allows them to share their production. They can import the means of production or final consumption goods from each other, and they can specialize in one or two sectors because they can give up part of their net product in exchange for the goods that they do not produce.

For the scope of this section, it does not really matter which redistribution mechanism we consider because it might be trade as well as a planned coordination mechanism. To be convenient for the countries involved, it is sufficient that the mechanism is successful in providing to each country amounts of commodities that are at least sufficient to cover the domestic demand that each country would have realized in an autarkic context.

In this paper, we analyse how the different specialization patterns of a group of countries may influence the environmental impact of carrying out productive activities. Considerations on how the institutional mechanism may redistribute the production among countries in a “satisfying” way are left for future studies.

Supposing that such a redistribution mechanism exists and that it is implemented, then the countries’ productive system is not bound to domestically produce the amount necessary for the satisfaction of domestic consumption demand. Consequently, these countries are free to specialize in different sectors. This also implies that the same global net product, which is the sum of the vector of the surpluses of the single countries, could be produced in many different ways.

Given that $y_c = [y_{1,c}, ..., y_{n,c}]$, where $n$ is the number of sectors, is the vector of the historically observed net output of country $c$, and the matrix of the Net Products $Y$ is the matrix of all the $y$ considered; that is, 

$$Y = \begin{bmatrix} y_1 & \cdots & y_m \end{bmatrix} = \begin{bmatrix} y_{1,1} & \cdots & y_{1,m} \\ \vdots & \ddots & \vdots \\ y_{n,1} & \cdots & y_{n,m} \end{bmatrix}$$

(2.1)

where $m$ is the number of countries.

The vector of the Net Total Product (NTP) is obtained by summing up all the sectoral net products of each country; that is, the elements of each
column of $Y$: see (2.2).

$$NTP = Y_L = \begin{bmatrix} \sum_{c=1}^{m} y_{1,m} \\ \vdots \\ \sum_{c=1}^{m} y_{n,m} \end{bmatrix} = \begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix}$$

(2.2)

Given a specific NTP vector, there are a huge number of matrices $Y$ that allow us to produce it.

Suppose that we use $L$ to denote the matrix of the direct and indirect labor associated with the subsystems producing the net output of matrix $Y$.

$$L = \begin{bmatrix} L_{1,1} & \cdots & L_{1,m} \\ \vdots & \ddots & \vdots \\ L_{n,1} & \cdots & L_{n,m} \end{bmatrix}$$

(2.3)

Using $l_{i,c}$ to denote the labor vector of the subsystem of good $i$ in country $c$, we have that $L_{i,c} = l^t l_{i,c}$.

The sum by columns of the matrix gives the country employment, while the sum by rows gives the global employment that is necessary for the production of the global sectoral net output. The sum of all of the entries of the matrix gives the global employment.

Furthermore, we denote with $CO2$ the matrix of the direct and indirect emissions associated with the subsystems producing the net output of matrix $Y$: see equations 1.6-1.7.

$$CO2 = \begin{bmatrix} CO2_{1,1} & \cdots & CO2_{1,m} \\ \vdots & \ddots & \vdots \\ CO2_{n,1} & \cdots & CO2_{n,m} \end{bmatrix}$$

(2.4)

The sum by columns of the matrix of CO2 emissions gives the individual countries CO2 emission, while the sum by rows gives global CO2 emissions divided by sectors (or commodities). The sum of all the entries of the matrix gives global emissions.

For each matrix $Y$ there is an associated matrix $L$ and an associated matrix $CO2$, which is determined through subsystems. The use of of subsystems is extremely powerful in this case. With the appropriate mathematical programming problem, it is possible to find a matrix $Y$ such that the sum by columns of the related matrix $L$ is fixed and determined by the original endowment of labor, while the sum by rows and by columns of the related matrix $CO2$ is minimized: see Appendix A.

Fig. 1 helps to visualize how the same NTP can be reached in different ways. Consider the graph on the left, see Fig. 1a. The graph shows a
three country example of a specific specialization pattern in the sectors of coal and iron. The segments below the shaded area are the national coal-iron frontiers for each country; that is to say, they represent the achievable combinations of net product of coal and iron of each country, while keeping the constraint on the labor employed fixed.\footnote{For a description of how to compute the national frontiers, see Boglioni and Zambelli (2016) and Boglioni (2017) in the case of mobility of the means of production, which is the case assumed in Fig. 1.}

Suppose that the domestic demand in coal and iron of the three countries is described by $y_1$, $y_2$ and $y_3$. As explained in Section 1, in autarky the countries could not divert their production from these points without lowering welfare: assuming that the domestic demand is satisfied in $y_1$, $y_2$ and $y_3$. The NTP would be represented by point $Y$, which, as explained in eq. 2.2, can be computed simply by summing up the net products in coal and in iron of the single countries.

If a redistribution mechanism of the kind explained above exists, the countries would be free to shift their production point to wherever it is more convenient. Suppose, for example, that their CO2 emissions would be lowered by shifting their surplus in $y'_1$, $y'_2$ and $y'_3$ as in Fig. 1b. As can be noted, point $Y'$ is exactly the same as the original. What changes is just the proportion in which each country produces the net product in coal and in iron. More generally, the number of combinations—specialization pattern—that...
allow us to reach point $Y$ is most likely a large number. This makes it possible, at least in principle, to realize the specialization pattern that minimizes CO2 emissions without having any consequence on the final demand and on the quantity of workers employed.

It might even be possible to increase the NTP while at the same time reducing CO2 emissions. The shaded area is called $SS$, which stands for Specialization Space because it describes the area in which point $Y$ can be moved with an appropriate specialization pattern. Suppose now that Country 2 could shift its production to $y_E^2$ and country to $y_E^3$ in Fig. 1b. The new NTP becomes $Y^E$, which is higher than $Y$ in both the goods considered. Actually, point $Y^E$ can be considered to be an efficient production point in the sense that in $Y^E$ the NTP of one good cannot be increased without increasing the NTP of another good.

All of the points inside the triangle $YQR$ represent points that are achievable combinations of NTP that improve in both coal and in iron. If the point is internal to the $YQR$ area, then it is possible to find another point that improves the net product in both the goods. If the point lies on the upper boundary of the $YQR$ area, then this is not possible because improving the NTP would imply a decrease in the NTP of the other good. The set of points that lies on the upper boundary of the $SS$ area is called the Net Product Possibility Frontier (NPPF)$^5$. As explained in Boglioni and Zambelli (2016) and Boglioni (2017), a Net Total Output vector with this characteristic can be reached when the countries considered exploit their CAs.

Moreover, point $Y^E$ has an additional interesting feature in that it is simply the original NTP represented by $Y$, multiplied by a factor greater than 1. That is, to say that both iron and coal are increased by the same proportion.

Fig. 2 has been constructed to clarify this point. Point $Y^E$ lies at the same time on the NPPF and on the ray passing through the origin and point $Y$. For all the points $Y^p$ that lie on segment $YY^E$ we have that

$$GS^p = \frac{Y_1^p - Y_1}{Y_1} = \frac{Y_2^p - Y_2}{Y_2}$$

that is to say, for all the points that lie on $YY^E$, the percentage improvement in Good 1 is equal to the percentage improvement in Good 2.

$^5$The set of points on the lower boundary of the $SS$ form the Inefficient Frontier (IF). This is called the Inefficient Frontier because it describes those points in which the available resources are exploited in the worst possible way. If the quantity of workers in the subsystems of coal and iron is kept fixed, then point $Y$ cannot go below the IF. For a detailed description of how the national frontiers, the NPPF and the IF can be constructed and their relations with CAs theory see Boglioni and Zambelli (2016) and Boglioni (2017).
Figure 2: The Gains from Specialization index. The graph shows how to compute the $GS$ index: see eq. 2.5.

Focusing attention on segment $YY^E$ is important for two reasons. First, the points belonging to it allow us, at least in principle, to redistribute to the single countries the additional surplus according to their preferences. The total produced net national product is along the ray greater than the actual historically realized production. A proper reallocation mechanism could allow for higher consumption by the countries. The motion along the ray represents points where the so called Pareto improvements are possible: see Boglioni and Zambelli (2016). Second, in these points it is possible to measure the improvements in the NTP with a scalar number instead of a vector. This scalar number has been called the $GS$—which stands for Gains from Specialization—index.

In summary, in a point like $Y^E$ we have that:

- CAs are exploited at best, so that the NTP vector is higher than the original;

- The NTP in both the goods considered is increased by the same proportion

When the NTP reaches a point like this, the gains from specialization index reaches its maximum and is called $GSF$.

Whether the CO2 emissions implied in a point like $Y^E$, in which CAs are exploited and the NTP is efficient, are higher than the originals depends
on the specific CO2 related to the subsystems in the original countries. This point is going to be developed further below for a case in which actual data is used.

3 The data

The data used for this paper are taken from the World I-O Database (WIOD). The WIOD provides $35 \times 35$ I-O tables for 40 countries for the period 1995–2011 and the related historical CO2 emissions for the period 1995–2009, so that the time span covered here is 1995–2009.

Among the 40 countries that are available, the 30 countries reported in Tab. 6 have been selected. The choice of excluding 10 countries is mainly due to their few dimensions or particular economic structure.

The number of sectors has been reduced from 35 to 17, and specifically to the primary and secondary sectors: see Tab. 7. We have done this because, as explained above, the underlying assumption in studying Net Product Possibility Frontiers and the related Specialization Space is that the goods considered can be standardized and exported. For this reason, the sectors included are those that enter the Standard International Trade Classification of the United Nations.

Another feature of this approach is that the NPPF are constructed while assuming that they work with real quantities, while the I-O tables are constructed by aggregating in industrial sectors a large number of goods through the use of market prices. The aggregation problem cannot be solved without having finely disaggregated I-O tables, but it is at least possible to deflate the I-O tables properly in order to take into account the fact that prices changes across countries and through the years. For a discussion of these problems and of the procedure adopted to deflate the I-O tables see Boglioni and Zambelli (2016).

4 CO2 emissions and specialization patterns: empirical results

4.1 The historical evolution of CO2 emissions

The use of subsystems allows a very clear and theoretically well defined notion of CO2 total—that is, direct and indirect—emissions associated to the production of individual commodities net output: see equations 1.6, 1.7 and 2.4.
A starting point of our analysis is the CO2 emissions per unit of net output associated to the subsystems. In other words, the sum of the CO2 directly and indirectly involved in the production of a certain net product computed using the notion of subsystems, as in 1.7, divided by the net output of the good under analysis.

The complete results are provided in the supplementary material. For reasons of scope, here we analyse a more synthetic index; that is, the weighted average of the subsystems CO2-emission per unit of net output. The weights are given by the quantity of labor employed in each subsystem, over the total quantity of labor of each country. The formula is provided in eq. 4.1

$$\overline{\text{CO2}}_{c,y} = \frac{\sum_{i=1}^{n} \frac{CO2_{i,c,t}}{y_{i,c,t}} L_{i,c,t}}{\sum_{i=1}^{n} L_{i,c,t}}$$

(4.1)

where $i$ identifies the sector, $c$ the country, $t$ the year, and $n$ is the total number of sectors.\(^6\)

The index $\overline{\text{CO2}}$ offers an overall view of the environmental impact of each country. The full results of this index are reported in Tab. 8 and Tab. 9, while Fig. 3 reports some examples for each area considered—that is, Europe, North America, Asia, and Others.

Fig. 3a shows the three countries of Germany, Italy and Poland, which represent Northern, Southern and Eastern European countries, respectively. The country that started with the highest emission was Poland, but it improves at a fast pace, especially during the 1990s. Italy and Germany also had a positive trend, even if they started from a low level. The graph also shows the results for Europe, which have been computed by summing up the I-O Tables and the related CO2 emissions of the 17 European countries that enter into this sample: see Tab. 6.

The Northern American countries all started from low levels compared to the countries of other areas, but Mexico and Canada successfully improved their emissions while the United States did not: see Fig. 3b.

Fig. 3c shows that in Asia, China was by far the worst country in 1995—more or less on the same level as Poland in Europe. China’s emissions then decreased constantly until the beginning of the 2000s, but then the trend stopped. Japan has low emissions and its trend was increasing until 2003, more or less, but then the trend reversed; consequently, there is little difference between the values in 1995 and the values in 2009. Overall, in “Asia”—that is, the five Asian countries reported in Tab. 6 considered as a whole—the emissions in 2009 are slightly lower than those in 1995.

\(^6\)If a country is a net importer of a good, then its net product is negative. Sectors with a negative net product have been excluded from the computation.
**Figure 3: CO2-emissions per unit of product.** The graphs represent the cross-sector average of the emissions per unit of net output computed as in eq. 4.1. The results for the full sample are reported in Tab. 8.

(a) Europe  
(b) North America  
(c) Asia  
(d) Others & the World

Finally, Fig. 3d reports the series for Australia, Brazil, and Russia, along with the data for all the countries considered as whole (i.e., the World). Russia and Brazil stayed stable, while Australia increased its CO2 emissions.

As the solid blue line shows, the overall trend for the World is slightly decreasing. There are some important countries—such as Japan or the United States—that did not improve sensitively, while others—such as Australia—even increased the emissions with respect to the $c_{CO2}$ index. However, it has to be noted that these countries started from fairly low levels when compared to other countries that reduced the CO2 emission.
Figure 4: The CO2-emissions isoquants. Each curve inside each area represents those combinations of net output that could have been produced keeping fixed the—minimized—quantity of emissions of CO2. The scale of each isoquant is $10^6$ kilotonnes of CO2.

4.2 CO2 isoquants

Fig. 4 shows two examples of NPPF and the related Specialization Space built on the basis of real data. This has been constructed to study those combinations that improve at the same time the NTP and the CO2 emissions. In Fig. 4a, the two sectors considered are Sector 9—“Chemical products”—and Sector 15—“Transport Equipment”—and the year is 1995. In Fig. 4b, the two sectors considered are Sector 1—“Agriculture”—and Sector 14—“Electrical and Optical Equipment”—and the year is 2009.

The vector NTP is represented by point $Y$. Inside the SS are drawn the “CO2 isoquants”; that is to say, those combinations of net outputs in which the minimum emissions of CO2, computed with the algorithm explained in Appendix A, are constant.

The values of CO2 reported are the values implied in the subsystems relative to the sectors considered of all the 30 countries in the sample, computed with equations 1.6, 1.7 and 2.4.

The thickest CO2-isoquant, which is called “Historical CO2 isoquant” in the figure, represents those combinations of net output that can be produced

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In this figure, the constraints on the other pollutants do not apply because they would have impeded to compute the minimum CO2 emissions for some points of the SS area, affecting the clearness of the Figure: see eq. A.15 of Appendix A. These constraints apply when all the 17 sectors are considered and the original NTP vector is kept fixed: see Section 4.3.
with an emission of CO2 equivalent to the historically recorded level. All of the points inside the \( SS \) that lie below the “Historical CO2 isoquant” can be produced with a quantity of CO2 emissions that are lower with respect to the historically observed ones.

It can be noted that the value of the CO2-isoquant passing through point \( Y \) in Fig. 4a is \( 0.94 \times 10^6 \) kilotonnes of CO2. Since the value of the “Historical CO2 isoquant” is \( 1.29 \times 10^6 \) kilotonnes of CO2, the figure shows that the original NTP in “Chemicals and Chemical Product” and “Transport Equipment” in year 1995 could have been produced with \( 0.94 / 1.29 \times 100 = 72.87\% \) of the original emissions implied in the two subsystems. If we consider Fig. 4b, the data suggest that the historically observed NTP in “Agriculture” and “Electrical and Optical Equipment” in year 2009 could have been produced with \( 0.71 / 1.34 \times 100 = 52.99\% \) of the historically observed emissions.

In other words, with the appropriate specialization pattern the historical NTP in the two sectors considered could have been produced with much lower CO2 emissions. Or, alternatively, by keeping the same level of CO2 emission—that is, \( 1.29 \times 10^6 \) kilotonnes in the case of Fig. 4a and \( 1.34 \times 10^6 \) kilotonnes in the case of Fig. 4b—it could have been associated with a higher NTP.

Please note that these benefits are reachable by considering just two sectors in each case. As shown below, when the number of sectors increases, the gains may be much higher.

All the combinations of net total output inside the area \( YQST^R \) in the case of Fig. 4a or area \( YQTR^R \) in the case of Fig. 4b represent those combinations in which:

- The net total output is improved in both the sectors considered with respect to the original one, using the same amount of resources and exploiting the CAs of each country;
- The CO2 emissions are lower than those historically recorded.

### 4.3 CO2 in an optimized framework: the case of Europe

As explained in Sections 2 and 4.2, the same NTP vector can be obtained through many different specialization patterns, each of which has a different environmental impact. With the appropriate algorithm, it is possible to identify the CO2 minimizing one and it may also be possible to investigate how to improve the NTP while at the same time reducing the CO2 emissions. However, this does not mean that the political conditions or that the markets would make this to realized in reality.
Figure 5: Optimized CO2 emissions. In the graph it is represented the ratio between the CO2-emissions in the optimal framework and the historical CO2 emissions.

In this paper, we apply the algorithm explained in Appendix A. We start from a sub-sample composed by the 17 European countries that enters into Tab. 6, which is used to study the CO2 emissions associated to the NPPF used in Boglioni (2017).

Focusing on European countries is also interesting because of the presence of common European institutions that might facilitate the adoption of environmentally-friendly policies. Therefore, Europe, as any other area with common institutions, is a privileged case for studying potential CO2-reducing specialization patterns. What distinguishes Europe from other areas is data availability and the political power of the common institutions that could determine coordinated policies.

Fig. 5 shows which would be the CO2 emissions in an optimized framework as a ratio of the historical CO2 emissions in Europe. The results are striking: they suggest that the CO2 might be reduced to 24%–28% of those actually produced.

It is important to note that these improvements in CO2 emissions would not worsen the emissions of other air pollutants because there are specific constraints in the algorithm that take all of the gases available from the WIOD into account: see Section A and eq. A.15 of Appendix A.

There does not seem to be any trend—either positive or negative. This
implies that there seems to be ample scope for coordinated policies to improve CO2 emissions through proper specialization processes and that there was no improvement in this sense during the years considered.

To study the achievable combinations of increased NTP and reduced CO2, it is useful to use the $GS$ index. As explained in Section 2, the *Gains from Specialization* or $GS$ index, is a scalar measure of how much the NTP of a group of countries could be improved thanks to the exploitation of CAs. The maximum achievable *Gains from Specialization* index, which represents the gains achievable when CAs are exploited at best, is called $GSF$.

Fig. 6 shows all of the possible combinations of $GS$ and—minimized—CO2 in 2005. In other words, it describes how the minimized CO2 changes as the “gains from specialization” pass from from 0 to their maximum level—the $GSF$—, which is represented by the vertical dotted line. The horizontal, thickest dotted line represents the historically recorded CO2 emissions.

In this case, the constraints of other air pollutants—that is, equation A.15—do not apply. Here we study possible trade-offs between CO2 reduction and improvements in the NTP thanks to a better exploitation of CAs. The insertion of additional constraints would imply the need to study many trade-offs, which would severely complicate the analysis and, therefore, this part is left for future studies.

As can be noted, the majority of the blue line stays below the historical emissions line. All of these points represent achievable specialization patterns in which the NTP is improved and CO2 is reduced. As can be noted, the historical emissions line is crossed when the $GSF$—which is 0.35—is very close. A $GS$ of 0.3 is achievable with CO2 emissions that are less than half of those historically observed.

This implies that the European NTP in 2005 could have been increased by 30%, while at the same time halving the CO2 emissions.

The case in Fig. 6 is just one of the 15 years considered, but the evolution is very similar in all of the years.

Fig. 6 also shows that the solid blue line has a slight slope for most of the domain of the $GS$ index, while it starts to grow rapidly and crosses the historical emissions line when the $GSF$ is close.

This is due to the fact that when the $GS$ is far from the $GSF$, the NTP vector is interior to the Specialization Space, and hence far from the NPPF: see Figures 1, 4 and 2. As explained in Section 1, this implies that there is a wide alternative of specialization patterns that realize a specific NTP vector, among which the one with the lowest CO2 can be chosen.

The closer that $GS$ gets to the $GSF$ and the NTP to the NPPF, the tighter the constraints of the minimization problem: see Appendix A. In other
Figure 6: Gains from specialization and CO2 in Europe. The graph shows the combination of gains from specialization (GS) and CO2 emissions which was possible to reach in 2005 through an optimized specialization pattern among European countries. The ascissa reports the distance between the historically determined total net national product vector and the net national production vectors obtainable when specialization and reallocation is occurring.

words, close to the NPPF the range of alternatives that allows to produce a specific NTP vector becomes narrower and, when the NPPF is reached and CAs are exploited at best, this range is extremely limited, we have practically only one specialization pattern, and in the majority of cases it implies that CO2 emissions are higher than the original ones.

Given that European countries were far from exploiting CAs during the period 1995–2009 (Boglioni and Zambelli, 2016; Boglioni, 2017), there seems to be a wide range of specialization patterns that could improve the NTP, while at the same time reducing the CO2 produced by the European countries.

An additional result is that since, as has been noted, CO2 emissions increase rapidly just when the GSF is close, even if the CAs worked well, it would be possible to achieve substantial gains in terms of CO2 emissions with little loss in terms of efficiency.
4.4 CO2 in an optimized framework, full sample

Extending the computation to the full sample, the spaces for reducing CO2 emissions and improving gains from specialization are even wider\(^8\). Fig. 7 shows the ratio between the minimized CO2 and the original emissions when the NTP is kept fixed. The supplementary material provides the tables that describe the specialization patterns that would allow to reduce CO2 emissions down to the quantities described in Fig. 7.

Between 1995 and 2001, an appropriate specialization pattern could reduce the CO2 emissions down to 20%–22% of the historical emissions. In the period 2002–2009, the percentage of emissions could have been even lower, with the lowest point in 2009, in which the emissions could have been less than 17%. Extending the sample, the benefits could have been even higher than those reachable in Europe.

This holds for both the environmental benefits and for the gains from specialization, as can be inferred from Fig. 8. The graph shows the achievable combinations of gains from specialization and minimized CO2 when

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\(^8\)As in the case of Europe, in the study of CO2-reducing specialization patterns under the hypothesis of a fixed NTP we apply the constraints on other air pollutant described in Section A. These constraints are not considered in the analysis of the relations between CO2 emissions and CAs.
Figure 8: Gains from specialization and CO2, full sample. The graph shows the combination of gains from specialization ($GS$) and CO2 emissions which was possible to reach in 1998 through an optimized specialization pattern.

In this case, the blue line is again far below the historical emissions line for most of the domain of the $GS$ but the maximum possible $GS$, the $GSF$, is much higher and passes from 0.35 in the case of the European countries to almost 1. This means that if the countries of the full sample exploited their CAs, the NTP could have been almost doubled in 2005. This also implies that with the full sample, 30% more of the NTP could have been produced with more or less 1/3 of the original CO2 emissions.

This evidence once again suggests that there is a wide scope for improving economic efficiency and the environmental impact of the economic activity. When more countries are involved, the potential gains deriving from CO2 minimizing and/or economically efficient specialization patterns will be higher.

4.5 Extreme scenarios, feasible objectives

Reaching the results described by Fig. 7 is not an easy task. As can be inferred from the tables reported in the supplementary material that describe
the CO2 minimizing specialization pattern, countries should specialize in a very few sectors—in most of the cases 1 or 2—in order to realize a CO2 minimizing specialization pattern.

This point can be highlighted using the labor mobility index, which is computed with eq. 4.2.

\[
\text{lm}_{i,c} = \frac{\hat{l}_{i,c} - l_{i,c}}{\sum_{i=1}^{n} l_{i,c}} \times 100
\]  

(4.2)

where \(l_{i,c}\) is the historically observed quantity of workers employed in sector \(i\) by country \(c\) and \(\hat{l}_{i,c}\) is the quantity of workers that should be employed in the same sector by the same country in the CO2-optimized scenario. It should be stressed that here the labor mobility is within the single country and not between countries; that is, migrations are not considered.

A positive \(\text{lm}\) index implies a sectoral inflow of workers and a negative sign implies a sectoral outflow. There are too many full results to be reported here and they are instead included in the supplementary material. Here, it is important to stress that according to these tables, most of the countries should concentrate all of their labor force in one or two sectors, shifting more than the 90% of their workforce and closing almost all of their industrial sectors. This seems to be too extreme a scenario to be achievable.

However, the scope for reducing CO2 emissions seems to be so ample that there is no need to realize fully the specialization patterns described by the tables on the labor mobility index to achieve a substantial reduction of CO2 emissions. This can be shown analysing the reduction in CO2 emissions that we could obtain by shifting a limited quantity of workers.

Suppose that the percentage of workers to be shifted is represented by \(x\), which goes from 0 to 100. Then, we can compute what alternative labor vectors in country \(c\) would look like if we shift a given quantity of workers in the direction suggested by the labor mobility index. This can be done with the following formula

\[
\text{l}_{c}^{new} = \text{l}_{c} + \frac{\text{lm}_{c}}{100} \times \text{L}_{c} \times \frac{x}{100}
\]  

(4.3)

where \(\text{l}_{c}\) is the vector of the labor employed in each sector by country \(c\), \(\text{lm}_{i,c}\) is the associated vector of \(\text{lm}\) indexes reported in the tables on the labor mobility index reported in the supplementary material, and \(\text{L}_{c} = \sum_{i=1}^{n} l_{i,c}\) is the total quantity of workers in country \(c\).
To each $l_{c}^{\text{new}}$ vector we obtain varying $x$ from 0 to 1, an associated vector $co2_{c}^{\text{new}}$ is computable in the following way:

$$co2_{c}^{\text{new}} = co2_{c} \cdot l_{c}^{\text{new}} \odot l_{c}$$  \hspace{1cm} (4.4)

Therefore, if the same fraction of workers to be shifted $x$ is applied to each country $c$, then we can plot possible combinations of workers to be shifted and achievable global quantities of CO2 emissions in a graph, such as the one in Fig. 9. In the case of Fig. 9, $x$ varies between 5 and 50. The graph shows the results in four years: 1995, 1999, 2004 and 2008. As can be noted, the differences from one year to another are slight.

**Figure 9: Different levels of CO2 emissions.** The graph shows how much the CO2 emissions could be lowered shifting a varying percentage of workers in the direction suggested by the $lm$ index.

The graph shows that in order to reduce CO2 emissions by more or less 5%, which was the aim of the first Kyoto Protocol, it would have been sufficient for each country to shift a little more than the 5% of their workforce. To reduce CO2 emissions by 18%, which is the objective of the Doha Amendment—the pact that should succeed the Kyoto Protocol—, countries would have to shift less than the 25% of their workforce.

\[\text{The symbol } \odot \text{ is the Hadamard product, which means that it is an element-by-element product. The symbol } \odot \text{ is the element-by-element division.}\]
It should be stressed that these results only refer to one year. Fixing a time span of 7–8 years—as in the case of the Doha Amendment—and supposing that technologies remain unchanged, it would be sufficient to shift more or less 4% of the workforce each year to reach the objectives. If technologies improve, this would shorten the time span and/or reduce the quantity of workers to be shifted.

**Conclusions**

The results of this paper suggest that the group of countries considered here are far from specializing in such a way as to minimize the environmental impact in terms of CO2 emissions.

We have constructed possible scenarios where specialization is possible. Our results are of a theoretical nature. We have investigated, given the information concerning CO2 provided by the data set we use, virtual reductions of CO2 and potential increases of the world surplus (NTP). Clearly, what we find is an upper theoretical bound that is based on the assumption that specialization is actually feasible. Whether this might be the case has to be investigated further. The importance of our work is that these potential upper bounds are computed.

What we find is that if these virtual or theoretical specializations had been realized, then the emissions during the period 1995–2009 could have been more or less 20% of those historically recorded. This is a very surprising and striking result. Further investigations on this point are, therefore, necessary and desirable.

Another important result is that these gains would not necessarily come at the expense of economic efficiency. Indeed, it seems possible to combine the gains from a better exploitation of CAs, and from the reduction in pollution due to CO2.

The principle of CAs suggests that the net output of a group of countries depends on their choices in terms of allocation of resources. One key assumption of the theory on CAs implies that the “invisible hand” of free markets will push countries who allocate resources in such a way that the overall net output will be maximized (Samuelson, 2001), while taking the quantity of resources and the state of technology of each country as given.

In this paper it is shown that if this assumption were verified and countries exploited at best CAs, then there would be a trade-off between economic efficiency and CO2 emissions. However, since there is poor evidence of the workings of CAs, there seem to be ample scope for exploiting them better, and hence increasing the net output while reducing CO2 emissions,
and without implying worker layoffs.

Moreover, it is interesting to note that even in the case where CAs are fully exploited, it would be sufficient to have a little loss in terms of economic efficiency to sensitively reduce CO2 emissions. This once again reinforces the idea that interventions aiming at preserving the environment through proper specialization patterns is an idea that deserves to be further investigated.

The approach adopted here is based on the computation of NPPFs, assuming that final goods and means of production can be traded across countries and it consists in using as benchmark a scenario that implies a strong restructuring of the economies considered and it might seem an extreme scenario. However, there is no need to implement fully the scenario described by the NPPF. As is shown above, even relatively small actions in the direction suggested by the NPPF may give a substantial contribution for contrasting CO2 emissions and global warming.

The databank used here requires a note of caution. In Boglioni and Zambelli (2016) it has been noted that the database might be improved because the I-O tables that are used for this study are fairly aggregated. More disaggregated tables would improve the precision and reliability of the computation.

The results presented here suggest that the scope for diminishing CO2 emissions through alternative specialization of countries are so ample that it seems worth collecting the proper data and further investigating the environmental implications of specialization processes.
References


Table 6: The list of countries*

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*The countries excluded are Bulgaria, Cyprus, Estonia, Lithuania, Luxembourg, Latvia, Malta, Romania, Slovak Republic, and Slovenia*
Table 7: The list of sectors*

1. Agriculture, Hunting, Forestry and Fishing
2. Mining and Quarrying
3. Food, Beverages and Tobacco
4. Textiles and Textile Products
5. Leather, Leather and Footwear
6. Wood and Products of Wood and Cork
8. Coke, Refined Petroleum and Nuclear Fuel
9. Chemicals and Chemical Products
10. Rubber and Plastics
11. Other Non-Metallic Mineral
12. Basic Metals and Fabricated Metal
13. Machinery, Nec
14. Electrical and Optical Equipment
15. Transport Equipment
16. Manufacturing, Nec; Recycling
17. Electricity, Gas and Water Supply

*The sectors excluded are Construction; Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Wholesale Trade and Commission Trade, Except for Vehicles and Motorcycles; Retail Trade Except for Vehicles and Motorcycles, Repair of Household Goods; Hotels and Restaurant; Inland Transport; Water Transport; Air Transport; Other Supporting and Auxiliary Activities; Activities of Travel Agencies; Post and Telecommunications; Financial Intermediation; Real Estate Activities; Renting of M&Eq and Other Business Activities; Public Administration and Defence, Compulsory Social Security; Education; Health and Social Work; and Private Households with Employed Persons.
Table 8: Cross-sectors average CO2 emissions. The Table shows the results of the index computed as in eq. 4.1, which is the cross-sector weighted average of the emissions per unit of net output of each country.

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Appendix A  The algorithm for the computation of a CO2 minimizing productive pattern

A key feature of the algorithm is that the CO2 emissions of a certain sector are supposed to be a linear function of the gross product of the same sector: see Section 1.

As stressed in Boglioni (2017), eq.ns 1.2-1.3 that are used to compute the vector $x_{i,c}$, which identifies the subsystem of good $i$ for country $c$, are all linear equations. Consequently, denoting with $CO2_{i,c}$ the total emissions related to subsystem $i$ of country $c$, computed as in eq. (1.7), we conclude that $CO2_{i,c}$ is a linear function of the net output $y_{i,c}$ and hence that

$$CO2_{i,c} = e_{i,c} y_{i,c}$$  \hspace{1cm} (A.1)

where $e_{i,c}$ is a scalar and it is a given parameter which may be called the environmental impact factor. It is convenient to organize the environmental impact factors in matrix $E$.

$$E = \begin{bmatrix} e_{1,1} & \cdots & e_{1,m} \\ \vdots & \ddots & \vdots \\ e_{n,1} & \cdots & e_{n,m} \end{bmatrix}$$  \hspace{1cm} (A.2)

Consequently, in matrix notation, we have that\(^{10}\)

$$CO2 = E \odot Y = \begin{bmatrix} CO2_{1,1} & \cdots & CO2_{1,m} \\ \vdots & \ddots & \vdots \\ CO2_{n,1} & \cdots & CO2_{n,m} \end{bmatrix}$$  \hspace{1cm} (A.3)

where matrix $Y$ is the matrix of the net national product—see eq. 2.1.

We want to find a new matrix $\bar{Y}$, such that the sum by column and by row of the related matrix $\bar{CO2}$ is minimized and such that

$$\bar{NTP} = \bar{Y} \geq NTP$$  \hspace{1cm} (A.4)

In a mathematical context, this is a typical linear programming problem. The objective function and the constraints of the problem are explained step by step.

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\(^{10}\)The symbol $\odot$ is the Hadamard product, that is to say that in $CO2$, each element in the position $ic$ of $E$ is multiplied by the element in the same position of $Y$, as explained in eq. A.1.
The objective function

To simplify the exposition of how to construct the minimization problem, suppose that we have three countries and three goods. We can reorganize matrix CO2 in a vector in order to obtain an objective function of the following type

\[ t_{CO2} = [CO2_{1,1}, CO2_{2,1}, CO2_{3,1}, CO2_{1,2}, CO2_{2,2}, CO2_{3,2}, \ldots, CO2_{1,3}, CO2_{2,3}, CO2_{3,3}]' \]  

(A.5)

\( t_{CO2} \) is the objective function of the minimization problem.

The NTP constraint

Matrix \( Y \) can be reorganized in the matrix

\[
y_{C} = \begin{bmatrix}
y_{1,1} & 0 & 0 & y_{1,2} & 0 & 0 & y_{1,3} & 0 & 0 \\
0 & y_{2,1} & 0 & 0 & y_{2,2} & 0 & 0 & y_{2,3} & 0 \\
0 & 0 & y_{3,1} & 0 & 0 & y_{3,2} & 0 & 0 & y_{3,3}
\end{bmatrix}
\]  

(A.6)

Basically, the problem is to minimize \( t_{CO2}'x \) with respect to \( x \), under the constraint

\[ YCx \geq NTP \]  

(A.7)

Solving this problem, we would find a vector of reproportioning factors \( x \) which, applied to the respective subsystems, would minimize the environmental impact keeping fixed or higher the quantity of goods produced by the three countries in an autarkic context. However, there are two more constraints that should be satisfied in our problem, they are, respectively, the labour constraint and the non-negative values constraint.

The labour constraint

The first is the total amount of work for each country, which is supposed to be fixed. We call \( L_{i,c} = i'L_{i,c} \) the total quantity of labour employed in subsystem \( i \) of country \( c \), while \( L_{c} = \sum_{i=1}^{3} L_{i,c} \) the total labour employed by country \( c \). Then, \( L = [L_{1,1}, L_{2,2}, L_{3,3}]' \) is the vector of total labour of the three countries. The labour employed in each subsystem can be organized in the following matrix

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The second condition to be respected is

\[ \text{LC} x = \text{L} \quad \text{(A.9)} \]

The non-negative value constraint

As explained in Boglioni (2017), when the means of production are free to move, this means that a country can run a deficit in specific sectors. This would generate negative subsystems. Intuitively, this should arise because, without further constraints, it may emerge the case in which a country imports more than it needs to run the production in the other sectors.

When this happens, by summing up the subsystems in order to obtain the system—bearing in mind the additive property synthesized by equation 1.5—, we obtain negative coefficients in matrices \( A \) and in vectors \( l \) and \( b \). Therefore, we have to introduce another to avoid this possibility. To do this, suppose that a generic element of subsystem \( i \) of country \( c \) is called \( s_{j,k,i,c} \), where \( j \) identifies the row and \( k \) the column of the subsystem.

What we want is that for every country \( c \), the sum of the \( n \) subsystems gives non-negative inputs and non-negative outputs. We denote with a minuscule \( a_{i,j,c} \), \( l_{i,c} \) and \( b_{i,c} \) a generic element of, respectively, the matrix of the inputs \( A \), of the labour vector \( l \) or of the gross output vector \( b \) of country \( c \).

Moreover, we identify with \( s_{i,j,k,c} \) a generic element of the subsystem \( S_k \) of country \( c \), which is defined as follows

\[ S_k = [A_k | l_k | b_k | co2_k] \quad \text{(A.10)} \]

Now, remind that

\[ a_{i,j,c} = \sum_{k=1}^{3} s_{i,j,k,c} \quad \text{(A.11)} \]

for each \( c = 1, 2, 3 \). In the Appendix to Boglioni (2017) it has been shown that if the elements \( s_{1,1,1,c} \), \( s_{1,1,2,c} \) and \( s_{1,1,3,c} \) have been determined such that the element \( a_{1,1,c} \) of country \( c \) is 0, then it necessarily follows that the elements \( a_{1,2,1}, a_{1,3,1}, l_{1,1} \) and \( b_{1,1} \) will also be 0, and the same argument applies to \( co2_{1,1} \). The same demonstration implies that if \( a_{1,1} >= 0 \), then all of the elements belonging to the same row will be non-negative.
Therefore, we just have to constrain the sum of the elements of the first column of each row to be non-negative in order to ensure that we will obtain non-negative $A$, $b$ and $l$ for every $c = 1, 2, 3$. If we define a matrix $SC$ as follows

$$SC = \begin{bmatrix}
    s_{1,1,1} & s_{1,1,2} & s_{1,1,3} & 0 & 0 & 0 & 0 & 0 & 0 \\
    s_{2,1,1} & s_{2,1,2} & s_{2,1,3} & 0 & 0 & 0 & 0 & 0 & 0 \\
    s_{3,1,1} & s_{3,1,2} & s_{3,1,3} & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & s_{1,1,2} & s_{1,1,3} & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & s_{2,1,2} & s_{2,1,3} & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & s_{3,1,2} & s_{3,1,3} & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & s_{1,1,3} & s_{1,1,3} \\
    0 & 0 & 0 & 0 & 0 & 0 & s_{2,1,3} & s_{2,1,3} & s_{2,1,3} \\
    0 & 0 & 0 & 0 & 0 & 0 & s_{3,1,3} & s_{3,1,3} & s_{3,1,3} 
\end{bmatrix}$$

(A.12)

The non-negative value constraint is that

$$SCx \geq 0$$

(A.13)

**Other air pollutant constraints**

As in the case of CO2—see eq. 1.6-1.7—, we can compute the emissions of a generic air pollutant of each subsystem. We denote the total emissions of a generic air pollutant of a subsystem with $AP$, so that $AP_{i,c}$ is the total emission in tonnes of one of the gases provided by WIOD, which are methane, nitrous dioxide, nitrogen oxides, sulphur oxide, carbon monoxide, non-methane volatile organic compounds and ammonia. The vector $ap$ contains the quantity of air pollutant related to each subsystem of each of the three countries, that is to say

$$ap = [AP_{1,1}, AP_{2,1}, AP_{3,1}, AP_{1,2}, AP_{2,2}, AP_{3,2}, AP_{1,3}, AP_{2,3}, AP_{3,3}]$$

(A.14)

Suppose now that the overall observed emissions of a generic air pollutant in our three countries-three goods example—that is, the total emission of the three countries considered together—is denoted with TAP, which stands for Total Air Pollution.

The additional constraint is

$$apx \leq TAP$$

(A.15)

and there is one constraint for each of the seven air pollutants reported by WIOD in addition to CO2.