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ON RICHARD GOODWIN'S *ELEMENTARY ECONOMICS FROM THE HIGHER STANDPOINT*

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1. Richard Goodwin's *Elementary Economics From The Higher Standpoint* (hereafter *EE* for short) is an extraordinary, original, still fresh and fruitful reading for the benefit of a vast variety of scholars, from undergraduates in the social sciences up to Nobel Prize winners in economics.

Published in 1970, just ten years after the 'revolutionary' *Production of Commodities by Means of Commodities* by Sraffa, *EE* should also be considered a 'revolutionary', albeit 'noiseless', book. With the former, in fact, it has many features in common: compactness and rigour, amongst others, which I shall try to bring out in the sequel. Moreover, the 'non-mathematical' structure which characterises *EE* throughout recalls what is explicitly written on the flap of Sraffa's book: 'it requires no knowledge of mathematics beyond elementary algebra'. This invites reflection on the following: that each of these two great economists of the last century (to whom Keynes might be added in this respect) felt the need at a certain point of his life to make recourse to 'simple' – though quite deep – tools, different from those which the tradition of the time imposed, for stating and re-stating non-elementary concepts and propositions. This becomes even more evident at a time in which certain sophisticated mathematical tools have long since been widely and intensively (and also improperly and unnecessarily on many occasions) employed in economic theory, perhaps far beyond the necessity of stating propositions with full rigour and intelligibility. However, the 'tools' of analysis (be they formal or conceptual) employed in science or in other disciplines are never *neutral*, inasmuch as they represent and interpret a 'system', physical or human, or physical *and* human at the same time, according to the specific circumstances of the case.

The formal approach pursued by the above-referenced authors seems to reflect, in fact, their strong willingness to convey a 'vision' of the world to a multitude of people far beyond the narrow circle of specialists, rather than use that approach as a purely pedagogical device. In the case of Goodwin, in particular, it should also be noted the special *form* he chose in presenting and discussing his propositions. What is peculiar in this case is not only the simple *geometry* of vectors he used, based as it was solely on the arithmetic operations of addition, subtraction and multiplication of vectors, but also the amazing *aesthetics* of almost all the 66 figures appearing in the book, as if he had been actually 'painting' a theorem, a process, or whatever else he was aiming at bringing forth or emphasising. The figures provide a visual and intellectual pleasure in following the reasoning they reproduce so concisely and, what is far more remarkable, their extreme essentiality and simplicity greatly help the reader in going straight to the point they aim at elucidating. In a sense, this recalls Hicks' definition according to which 'Economics is more like *art* or philosophy than science'. (Hicks, 1983, p. 4, italics added)

EE, therefore, should now be considered a 'classic', in the full sense of the word, in the economic literature, as I will endeavour to show in the following sections.

2. The intellectual environment surrounding economic theory at the time in which *EE* was written and published was characterised by at least three important events: (i) a lively debate in capital theory between the two Cambridges across the Atlantic; (ii) the recent accomplishment of the First Neo-Classical Synthesis, and (iii) some early signs of what later became the Rational Expectation devastating tsunami in economics.

Like any other book, *EE* could be read from various perspectives, according to the preferred key reading one chooses to adopt. My brief reflections here will adopt that which I consider most suitable for emphasising certain aspects of Goodwin's 'philosophical' standpoint in economics which might have been somewhat overlooked in his otherwise very well known literature. On this occasion, I re-read *EE* with different eyes than those I used when I read it for the first time as a student at Cambridge. And, having had the honour as well as the privilege of being a student of him, I am also able to add, here and there, some bits of personal memories on specific points which came out of conversations with my former tutor.

Three features of the Preface are worth mentioning openly from the very beginning:

(i) the *topics* selected,
(ii) the *people* explicitly mentioned,
(iii) the *intention* of the book as being "a guide for the perplexed".

The topics selected and tackled establish immediately and succinctly the general perspective Goodwin is taking up: global behaviour, interaction of the parts, dynamics. These are shown to be strictly interconnected, each one properly looked at in the light of the others. Furthermore, the topics reveal an updated perspective of modern economic theory, going back to the Classical economists, from Adam Smith's *Wealth of Nations*, through Ricardo's general view of the *Political Economy* up to Marx's reproduction schema of *Das Kapital* and then passing through the outstanding and evergreen contributions of Wicksell, Schumpeter and Keynes.

Among the eight people explicitly mentioned in the Preface we finds von Neumann and Leontief, for their having aptly made sense of Walras' general equilibrium 'empty box', and Ramsey, for his having re-cast the problem of capital accumulation 'in a lucid and fruitful way', p. ix.

The intention explicitly expressed by Goodwin right from the beginning is that of being 'a guide for the perplexed through a part of the weird and wonderful world of modern economic analysis', p. x. Stated in these terms, it finely evokes 'The Guide of the Perplexed' by Moses Maimonides, the 12th-century philosopher, who wrote the book for those scholars who were confused by the conflict between religion and the scientific and philosophical thought of the day.

> However, even when one who truly possesses knowledge considers these parables and interprets them according to their external meaning, he too is overtaken by great perplexity. But if we explain these parables to him or if we draw his attention to their being parables, he will take the right road and be delivered from this perplexity. This is why I have called this Treatise "The Guide of the Perplexed"'. (Maoimonides, 1963, p. 6)

3. The general framework set up in *EE* has the following distinctive characterizations.

The first of these, and the most crucial, is the *interdependence* of the parts composing the economy. This marks a huge distance from the *cet. par.* approach of Marshall, but also from the too severely aggregative approach of Keynes. To make the exposition as simple as possible and to use the two-dimensional space for the geometrical representation, the economy is supposed to be composed by only two sectors, each producing one single output. Goodwin carefully makes it evident that '[t]he basic concepts applicable to the more complex reality of many goods will usually apply to the simple case of two goods, where their nature is clearer and easier to grasp. The converse is less valid, […]', *EE*, p. 1. The latter statement is neither so obvious nor redundant as it might at first appear. In fact, the transition from two to three or more cases has in many circumstances marked a point of discontinuity. For example, in economics, Harrod (Harrod, 1961, p. 783-784), has naively believed that in Sraffa's book the exchange values of the commodities in terms of one another were determined by the ratio of the excess of production of any commodity *a* to the excess production of any other commodity *b*. Harrod's misunderstanding was due to wrongly extending what is specifically valid only in the two-industry case in a self-replacing state to the more general case of three or more industries (or even to the two-industry case in a *non*-self-replacing state) – and in this way perilously echoing the principle of the demand-and-supply determination of prices. This was one of the rare occasions, after the publication of his book, in which Sraffa felt compelled to come out of his traditional academic silence by publishing a comment on Harrod's review of his 1960 book showing Harrod's misunderstanding, (Sraffa, 1962).

A more striking example comes from mathematics: in the 17th century the question was

raised as to whether for a given integral exponent $n > 2$ the equation $a^n + b^n = c^n$ can be solved with positive integers a, b, c. As is well known, Fermat's Last Theorem states that that equation is *not* solvable in integers for any $n > 2$ – being instead solvable only for $n = 1$ and $n = 2$. The proof of this theorem was given many years after Fermat's conjecture.[1]

To further underline how careful Goodwin's above statement is, it is worth noticing that it is not by chance that in his 1953 paper (Goodwin, 1953) he uses throughout *three*-industry matrices for giving examples of the general statements therein contained.

In the Classical fashion, the core of the economy is made to coincide with the productive sphere – this being the second important characterization of the general framework. It is here, in fact, and not elsewhere, that the crucial decisions shaping the economic structure are taken as to what will be produced, how and at what prices, 'and most important of all, how much each consumer is given to spend on consumption', *EE*, p. 2. As a consequence, the first elements to be considered are the methods of production available in the economy, represented by a set of given *and constant* coefficients of production – hence the widely simplifying assumption of constant returns to scale. In nicely explaining the distinct meanings of *linearity* and *homogeneity* of the 'production function' thus assumed, he correctly states that it was 'first used by Wicksell in economics and used and misused by countless economists since'– a statement which could not sound more appropriate in the present time. On the other hand, no mention at all is made of Cobb and Douglas, as would have been quite natural to expect at this juncture, in line with the diffuse, although incorrect, general belief. Although this is not the only one case in which the Goodwin's Schumpeterian roots make their appearance, the remark made by one of the reviewers of *EE*, stating that 'Goodwin acknowledges a debt to Schumpeter, as well, but its nature is difficult to locate from the text' (Bronfenbrenner, 1971, p. 463), seems off the point.

The third important characterization is the immediate introduction of 'dynamics' and the extremely elementary cases considered at first. As will be made evident in the sequel, Goodwin's 'dynamics' is of a very special type. Apart from its analytical and conceptual aspects, it is far more worth noticing here the essential reason behind his choice of treating the system dynamically from the very beginning: as he will say explicitly later in the book, 'it is only by observing *change* that the interconnections of a system can be found', *EE*, p. 161, italics added. This will be in fact one *leitmotiv* appearing throughout *EE*. (It should be noted, however, that the inference problem referred to now is unsolvable, except for very special systems. This is why he decided to decouple interconnections and study change in smaller systems that could be analysed as planar dynamical systems.)

4. Within the space of only four pages and with the aid of only two diagrams, (*EE*, pp. 7-10), Goodwin succeeds in providing the preliminary basic concepts and definitions required to put the reader in the condition to follow even the most complicated reasoning. After having emphasised the *asymmetric* duality of quantities and prices – the former could exist without the latter, but not *vice versa* – he nonetheless makes it clear that the role of prices is instrumentally crucial, not so much for trivially allowing comparability between otherwise incomparable quantities by means of their reduction to scalars, but rather because *decisions* and *valuation* are made concretely possible.

To get a visual reference of the diagrams Goodwin employed in *EE*, figure 1 gives the simplest representation of the economy with its basic variables. (Later on in the paper, a couple of diagrams will be used along the lines of Goodwin's, just to give some of their flavour.) By writing $\mathbf{p}$ as the price vector, $\mathbf{a}_i$ as the input per unit output vector of sector i, $\mathbf{e}_i$ as the unit gross output vector of sector i, $\mathbf{u}_i$ as the unit net output vector, certain definitions and concepts and their geometrical representations can easily be given. For output 1, for example, produced with methods of production $\mathbf{a}_1'$, cost per unit output is ob – oc, revenue per unit output is ob, surplus

or profit per unit is o*c*, profit rate (π) is o*c*/*cb*. Due to the assumption of constant costs, the profit rate will be independent of the scale of production and of the level prices. In addition, a convex set of methods of production is assumed.

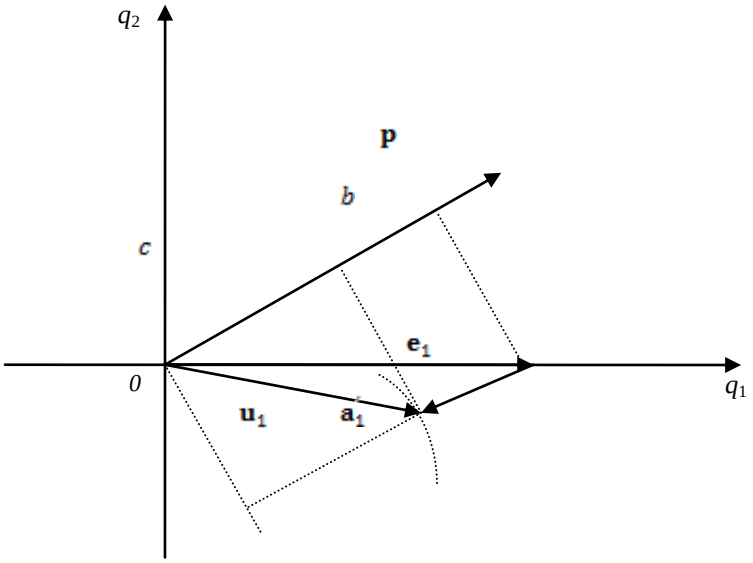

Fig. 1

Equipped with the simple conceptual and geometrical framework above, the reader is allowed to enter first in an imaginary closed economy, with no durables, no technical progress and each sector producing one single output. It is imaginary, however, not so much because of these assumptions only, but rather because labour, though necessary for production, is supposed to be available in such a plentiful quantity that no price will be paid for it. This assumption – which is coincident with that made by Sraffa (Sraffa,1960, p. 12 and p. 94), on the basis of a suggestion of Marx for calculating a maximum rate of profits corresponding to a zero wage – seems at first only a mere simplifying device, which indeed it is in the first instance. However, more to the point, this very assumption also allows Goodwin to state in the most appropriate way the notion of the 'viability' of a system – as will be seen presently. Also, the term 'factor of production' is *exclusively* reserved for *un*produced input (like labour and natural resources). On further reflection, in effect, since the root of the word 'factor' actually comes from the Latin word 'facere' (to make), only labour should properly be defined as a 'factor', being the only input capable of 'making' something – a feature which is not possessed by any produced means of production whatsoever. (An exception, perhaps, can be made for the self-reconstructing and self-reproducing Turing Machines.)

Within this imaginary world, the problem to be solved is quite a complex one, for it consists in *actually* determining the amount of each output to be produced, by which methods, and at which prices goods should be exchanged. It is a fantastically titanic enterprise to undertake *in practice*. In this connection, Goodwin refers to Walras' *tâtonnement* – the name being felicitous, as he quite aptly says, but at the same time also 'ominous', for 'it refers to the search, by tapping, of a blind man for a goal', *EE*, p. 12. This very characterization will be at the basis of Goodwin's own exposition of the process referred to above, and also it will help acknowledging, at the same time, its profound and rich value.

5. The Walrasian *tâtonnement* is an intriguing and not conceptually so simple a process as it might appear in the first instance. In two distinct papers, Goodwin had already neatly clarified how Walras' device should be interpreted.

> [Walras] explicitly states that [the *tâtonnement*] is only a mathematical method of solution and not the practical one exemplified in the behaviour of real markets". (Goodwin, 1951, p. 5)

> [Walras] thoroughly confused two related but distinct questions: the question of the existence of a solution and how to find it, with the question of the reality of a dynamical process and its stability. At some points he implies that his treatment bears only on the first and at others he clearly relates it to the second. I incline to the opinion that he wished merely to show the existence of a solution and to assert that this is the actual one realized, but *not* that the hypothetical process by which we might discover this solution is the same as the motion by which an actual economy reaches equilibrium. (Goodwin, 1953, pp. 59-60)

In *EE*, Goodwin seems to follow the same procedure of his 1953 paper, which at that time allowed him to resolve 'the vagueness of the Walrasian apparatus by resolutely embracing both horns of the dilemma', which meant *assuming* 'that the trial and error solution of the Walrasian equations reproduces step by step the process by which an economy evolves in time'. (Goodwin, 1953, p. 60) This looks to be only a compromise to make things workable, because he always had very neatly in mind the distinction between the *practical* solution and the *mathematical* one or, to put it another way, between the *actual* dynamics and the *virtual* process.[2]

In effect, this very neat distinction, much emphasised by Goodwin, hides a far more profound methodological insight. The image of a *blind* man in search of a goal by tapping represents an excellent approximation of the analogous searching activity by any human being in the real world, with the consequence that the goal might be found immediately, or after a considerably long lapse of time, or *not at all*.

There is another noteworthy aspect of Goodwin's approach to the Walrasian *tâtonnement* which, thanks to the many enlightening works of Velupillai, (Velupillai 1989, 2000, 2002, 2005, 2010) can now be seen very clearly: he seems to have been interested in showing *how* it was possible to arrive at the solution, and in so doing he was very careful in the *construction*, step by step, of the process eventually leading to the solution. This methodology, not yet completely defined perhaps in Goodwin's own framework, must be sharply contrasted to that generally followed in such circumstances, the latter methodology arbitrarily dividing the *existence* of a solution from the *method* through which it might possibly be found. As Velupillai has convincingly stated with extreme clarity:

> The von Neumann paper of 1928 (von Neumann, 1928), introduced, and etched indelibly, to an unsuspecting and essentially non-existent Mathematical Economics community and tradition what has eventually come to be called '*Hilbert's Dogma*' […], 'consistency ⇔ existence'. This became – and largely remains – the *mathematical economist's credo*. Hence, too, the inevitable schizophrenia of 'proving' existence of equilibria, first, and looking for methods to construct them at a second, entirely unconnected, stage. (Velupillai, 2011, p. 91)

In *EE*, Goodwin rightly indulges in a meticulous description of the working of the iterative method (pp. 13-19) as a means to emphasise the *constructive* (and perhaps also the *intuitionist*) aspects of the process. In a sense, to paraphrase Velupillai's felicitous expression (Velupillai, 2011, p. 95), the *existence* of an equilibrium is seen by Goodwin more *as construction*, rather than as a separate question to be settled independently of the method by which the economy

would eventually arrive at that equilibrium.

6. The very first pages of *EE*, like those of Sraffa's book, present very important and concise ideas that will be found disseminated throughout the whole volume.

By chapter 2, in fact, at least one important aspect of Goodwin's methodology seems definitely settled: 'understanding' means going through *change* and interpreting how and why it takes place. Within the static framework considered, he sees the market acting as a computer and the price-market mechanism as a medium for conveying the required information to solve the problem of how much to produce, how to produce, and at what prices. The 'computer' is of course of a very special kind, for it embodies 'human beings amongst the linkages', *EE*, p. 12. What is of most interest here is the *process* which is taking place, the temporal and logical *sequences* of the reactions, the different types of dynamic adjustments. By going through the literature, Goodwin traces out and selects five types of dynamic adjustments, according to the different profiles of price and quantity reactions – the latter type being the 'cross-field dynamics', which combines, so to speak, the Walrasian and Marshallian adjustment reactions to profit differences.

The starting point is a situation defined as 'primitive chaos', because nobody knows anything, except the extant methods of production. No information about prices is available, and therefore it would be impossible in these conditions to fix any mark-up whatsoever. The strategy is then to start with an arbitrary set of prices with a corresponding arbitrary set of mark-ups. Yet, as each of these arbitrary mark-ups would be fixed by each single producer independently of the others, they would as a whole result simultaneously 'invisible' to all producers. On the contrary, a rate of interest fixed between the highest and the lowest mark-up in a loan market or, what turns out to be the same, by a state or central bank, would be a simultaneously 'visible' magnitude for everyone. In this way a definite reference point would exist, thereby enabling the 'trial-and-error' process to begin.

The arrival onto the scene of a rate of interest on loans and the different ways in which it can be fixed has a long tradition, starting with a well-defined form in Marx's *Das Kapital* (Marx, 1971, p. 365) passing through the very well known Wicksellian duality (Wicksell, 1965, ch. 8) up to Sraffa's suggestion for the closure of the system (Sraffa, 1960, p. 33). In all these cases, the institutional credit arrangements *outside* the system of production turn out to be crucial in the determination of the income. In Goodwin's framework the rate of interest serves, in the first instance, as the pivotal variable through which the adjustment process, which ultimately leads to the price vector that allows for the simultaneous attainment of the common maximal profit rate for the two sectors, can effectively proceed.

It is worth looking more closely at this 'step by step' process as it is described in *EE*, not only to admire the simplicity and the logical strength of Goodwin's own description, but also to bring to the fore the far more important methodological aspects of the process referred to above.

He begins with an arbitrary unit price vector $\mathbf{p}'$ on which each producer *i* chooses the method of production $\mathbf{a}_i'$ yielding the highest profit rate π_i. The profit rates are obviously different from each other, and an arbitrary common rate of interest r' fixed from outside the system of production makes such a difference clear. The reaction will be, given the chosen methods of production, to raise the price of the lower profit rate sector and to lower the price of the higher profit rate sector, thereby establishing a new price vector $\mathbf{p}''$. In this way the difference between the two profit rates will be correspondingly reduced, but not eliminated. This creates the condition for fixing a new rate of interest r'', though within a range which has been reduced by the actions taken in the preceding step. This process will go on until the price vector $\mathbf{p}^*$ is *found*, and the methods of production $\mathbf{A}^* = (\mathbf{a}_1^*, \mathbf{a}_2^*)$ are *chosen*, such that the difference between the two profit rates has been reduced to zero and the two sectors will therefore

simultaneously yield a common maximal profit rate $\pi^* = r^*$.

It is worth noting that this is one of the cases (which will not be unique, as will be seen in the sequel) in which Goodwin makes in passing a lapidary and profound remark by simply and concisely stating that '[t]his determination of prices and methods of production is 'best' only for profits and in itself implies nothing for the community', *EE*, p. 16. This is one of the examples in which a political or an ethical judgement is being expressed, to signify and to remind us, discreetly but firmly, that economics cannot avoid at some point such types of judgements. Another explicit example of this sort will be encountered in certain problems of dynamic programming – as will be seen later on in the paper.

It is also well worth noticing, in the last sentence already quoted and in the context referred to now, the word 'community'. This word – which is a very rare fish in the individualistic-based theoretical economics – has a strong and not at all banal meaning, to signify a people whose members are supposed to be linked together by at least some socially-based common interests. This vision should be contrasted to that which is typical of the basic neoclassical paradigm, which contemplates instead an assembly of self-interested 'monads'.

The above 'step by step' procedure can therefore be summarised in a schematic way as follows:

(1) $\mathbf{p}'(1) \to \mathbf{A}'(1) \to \pi'_1(1) \neq \pi'_2(1);\ \ r'(1) \in (\pi'_1(1), \pi'_2(1)) \to r'(1) = \pi'(1)$

(2) $\mathbf{p}''(2) = \mathbf{A}'(1)\mathbf{p}''(1+\pi'(1)) \to \mathbf{A}''(2) \to \pi''_1(2) \neq \pi''_2(2);\ \ r''(2) \in (\pi''_1(2), \pi''_2(2)) \to r''(2) = \pi''(2)$

(3) $\mathbf{p}'''(3) =$ ……

……

(n) $\mathbf{p}^* = \mathbf{A}^*\mathbf{p}^*(1+\pi^*);\ \ r^* = \pi^*$

where the numbers within brackets stand for the number of the trials made.

It is instructive to have a look at the corresponding *visual* counterpart of the above procedure, which easily and smoothly demonstrates the essence of the entire process as well as some other interesting properties. By looking at figure 2, in fact, had the initial price vector (as, for example, either $\mathbf{p}_s$ or $\mathbf{p}_t$) happened to be very close to the unit output vector $\mathbf{e}_i$ (i = 1, 2) the profit rate of sector i would be very large compared with the profit rate of the other sector, thus inducing (graphically speaking) the initial price vector to rotate away from $\mathbf{e}_i$. Therefore the clockwise and the counter clockwise movement of the initial price vector (respectively, either $\mathbf{p}_s$ or $\mathbf{p}_t$) portrays the idea of *how* the searching activity engaged in by the producers and by the central bank (or the state) should take place in order to find the 'best' position of the system. It is to be noted from the diagram that vectors $\mathbf{p}_s$ and $\mathbf{p}_t$ have been chosen in such a way that they represent the maximum level achievable by price 2 and price 1, respectively, for making possible the production of good 1 and good 2, respectively, without loss.

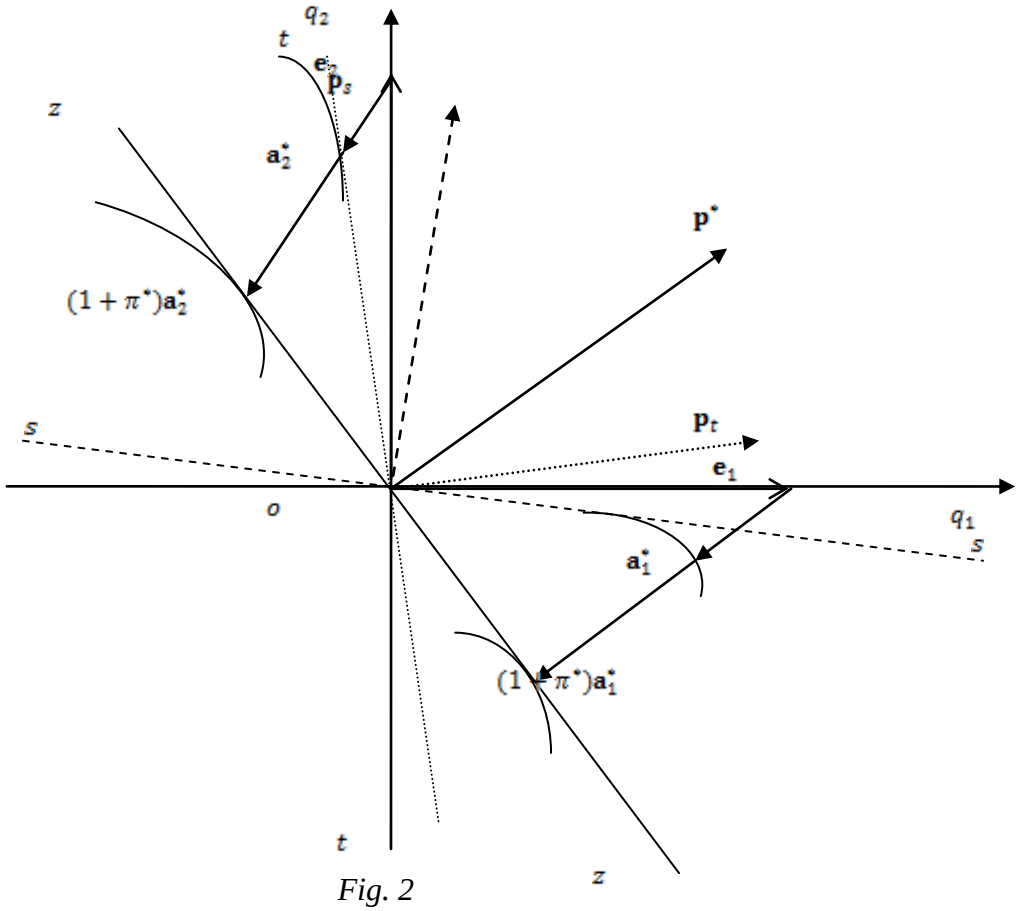

Fig. 2

The procedure would not be so different had the producers chosen different methods of production.

The same diagram can be used to visualise what Goodwin calls 'the fundamental theorem of the economic calculus', that is 'for any given gross output whatsoever the largest net value product will emerge from the profit maximising choice of technique'. (*EE*, pp. 27-28) This result, which can be defined a *social* optimum within the imaginary world with labour paid a *zero wage*, is considered by Goodwin a remarkable conclusion, in so far as each producer seeks to maximise profit but no one is maximising the aggregate net value product, and it can be straightforwardly connected to Adam Smith's long debated 'invisible hand' and *laissez-faire*. However, as Goodwin perspicaciously notes:

> It has often been used to justify private property and capitalism but in fact is politically neutral, being just as, or perhaps more applicable to a socialist or communist society. (*EE*, p. 28)

Later in the book, in dealing with decentralised planning, Goodwin comes again to this central theoretical topic in connection with the optimal combination of prices, output and techniques:

Though it *exists*, actually to *realize* it is an altogether different and more difficult task. The complicated question of stability arises: will this optimal solution be attained under specified conditions, and, if realized but being displaced, will the optimal solution be regained? A related question is: can we, by computer or otherwise, *calculate* the optimal solutions? (*EE*, p. 40, italics added)

7. By employing figure 2, the notion of the 'viability' of a system can be easily obtained from a glance at figure 3. Having assumed a zero price for labour, Goodwin's notion of viability turns out to be perfectly compatible with Sraffa's, (Sraffa, 1960, pp. 3-5), for it can easily be accommodated to incorporate workers' subsistence along with the means of production, and thus it can also handle the case of a *zero* surplus system.

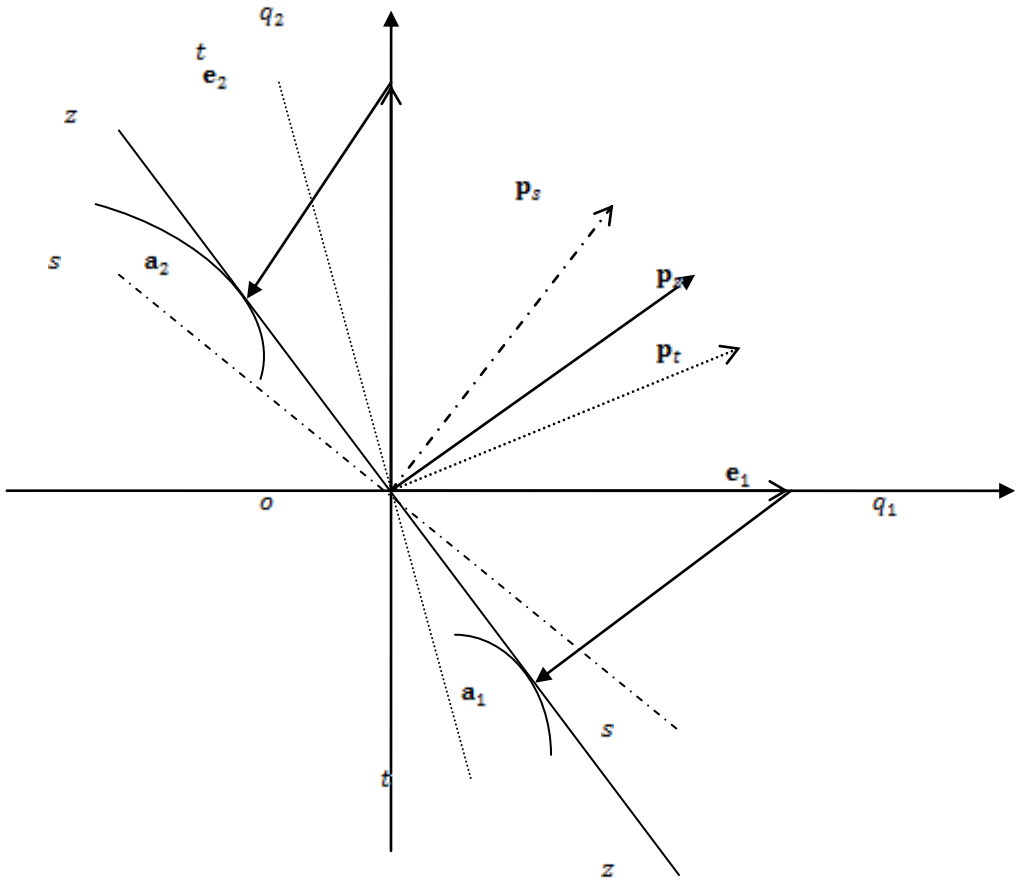

Fig. 3

In figure 3 the line *z* passing through the origin and perpendicular to the unique price vector $\mathbf{p}_z$ (which makes possible the reproducibility of the economy) is tangent to the isoquants, and therefore there is no surplus. A different price vector, either $\mathbf{p}_s$ or $\mathbf{p}_t$, would simultaneously create a surplus in one sector and a deficit in the other, and the system would be non-viable.

Both notions are different from that originally given by Hawkins and Simon, 1949, whose condition necessitates instead of the existence of a *positive* surplus, because it excludes workers' subsistence as a constraint.[3]

To have just an idea of the clarity and terseness of Goodwin's presentation of the notion of viability, a comparison with the analogous discussion in DOSSO would be illuminating, where the Hawkins-Simon condition is discussed. (Dorfman, Samuelson, Solow, (1958), pp. 212-221) In DOSSO, the discussion takes several pages full of equations (albeit very simple) and numerical examples before the authors get to the point.

8. Many pages of *EE* are devoted to dynamics, one of the central topics of Goodwin's original contribution to theoretical economics.[4] From the very beginning, starting from chapter 2, the essential character of a system to structurally 'oscillate' is beautifully presented with extremely simple but nonetheless comprehensive diagrams (*EE*, pp. 19-23), and, by contrast, the apparently attractive notion of 'equilibrium' and of its supposed stability seems suddenly to lose all its force or, at least, its own long standing centrality.

The dynamics of the economy is also dealt with in several central parts of the book, especially when problems of accumulation in a planned and in a market economy are widely discussed in a model involving durable goods, as in chapter 5, or when the wage-price inflationary process and the cycle generation by stocks-output reaction are briefly but efficaciously illustrated, as in chapter 6.

Within these topics, most of which are characteristic of many of Goodwin's original contributions to theoretical economics, the von Neumann model obviously had to find a place. The gist of this seminal model is easily explained by means of only one diagram (*EE*, p. 30) and the reader is told *how* to 'discover' the maximum growth rate (the von Neumann eigenvalue):

> extend progressively the unit isoquant until their common tangent passes through the origin. This gives the locus of all possible inputs for the expanded scale of output, $(1 + g^*)\mathbf{a}$, and the input which leads from **q** back to the origin, is the required solution, since all last year's outputs are exactly used up by this year's inputs. (*EE*, 32)

From this, the essence of the optimal consumption and the optimal growth are smoothly and easily explained.

However, Goodwin will come back again to von Neumann's model later in the book to make some 'political' and 'ethical' comments on it, which – within the already vast literature of the time – are quite unusual, if not almost impossible, to find.

> In spite of the dazzling brilliance of the von Neumann analysis, – Goodwin writes – an analysis which has helped to set the whole generation of economists to rethinking basic concepts, there are some unsatisfactory aspects. This growth which is maximized is simply the most rapid proliferation of near misery, as if one were to set about unrolling an endless slave economy on a deserted continent. This is growth in a very narrow sense and it can be doubted whether such growth is desirable. What has attracted economists to von Neumann is not the details of his model but the higher power of his theoretical apparatus. (*EE*, pp. 85-86)

This very last reflection, *mutati mutandis* and with the obvious *caveats*, might be extended also to most economists who had been working on Sraffa's *Production of Commodities by Means of Commodities*.

If Champernowne's paper (Champernowne, 1945) has had the merit of having translated into plain language the meaning of the mathematical intricacies of von Neumann's paper, it might be said that Goodwin succeeded not only in rendering them analytically digestible by preserving, at the same time, their essential rigour and meaning, but also to offer a new perspective for reading and interpreting that model.

9. A comparison between a free market economy and a planned economy regarding certain aspects of their respective functioning may seem an old fashioned exercise today. Nonetheless, it still proves useful in that it helps in bringing to light some fundamental mechanism of the functioning of the contemporary market economies.

This is precisely what happens on reading the pages of *EE* dedicated to a sketch of 'disequilibrium dynamics'.

Within this framework, Goodwin makes an ironical but firm criticism of the well-known Say's Law. Although the national accounting system on the one hand and the prolonged discussions on the economics of Keynes and on Keynesian economics on the other have been the occasion to clarify over the past years most aspects surrounding that law, its incumbent presence is still averted here and there in the economic literature.

Goodwin rightly rejects the two basic assumptions for which Say's Law is supposed to be generally valid: (i) all incomes are spent on goods (ii) their spending coincides in time with the sale of the net output which gives rise to those incomes.

> This venerable 'Law' is no law since not only is it not invariably true, it is never true because both assumptions are false. (*EE*, p. 38)

In a very simple diagram (*EE*, p. 39), he shows the locus of possible demands of gross output geometrically and as a consequence, among the many of these, he also shows the *particular* case in which 'equilibrium' might be achieved. Therefore,

> Say's Law is wrong in many ways, but its most profound, most fundamental, wrongness lies in the fact that expenditures connected with production do not coincide with finished output. (*EE*, p. 45)

With this criticism at the background, he is as a consequence in the most appropriate condition of affirming that 'Say's Law provides no comfort for the free market economy' (*EE*, p. 47). One of the bases for such a statement is provided by the running down or the piling up of stocks, which in a free market economy cannot be stopped nor be reversed, but rather can only be worsened – a case christened by Goodwin as 'a primitive form of 'knife edge'', which, in his own conception of how a capitalist economy actually works, is seen from the perspective of a 'cycle':

> The private entrepreneur can only invest if he knows, or thinks he knows, what other entrepreneurs will invest in the future. This leads to self-fulfilling prophecies of gloom or glory. The history of capitalism has fully exemplified this with alternations between buoyant demand, high profits, considerable inflation and depressed markets, unemployment, low profits. (*EE*, p. 47)

By contrast, in a planned economy the same running down or piling up of stocks will convey the 'right' information to the planner to decrease or to increase, respectively, the rate of expansion, thereby moving the system towards equality between demand and supply.

10. When labour is being paid a positive wage, other problems can be dealt with and most of the variables in the diagrammatic representations must be redefined per unit of labour.

A stationary economy is considered first. Within Goodwin's framework this particular structure of the economy turns out to be instrumental to putting to the fore apparently standard arguments through a non-standard form enriched by insightful reflections.

The stationary economy is supposed to be functioning in a competitive regime. As a consequence, by starting from any arbitrary profit rate and set of prices, the producers will

choose those methods of production which allow them to pay the highest wage rate. The competitive structure, however, will induce prices to change in such a way as to bring about a uniform wage rate. Exactly the same result would have been obtained had the wage rate been fixed instead of the profit rate. In either case, the Classical-Sraffian characterisation would emerge: income distribution is completely arbitrary, which means there is the possibility of attributing *any* value to either the wage rate or the profit rate from *outside* the system of production.

This being established, the experimentation of starting from a virtually zero profit situation and then going up to the maximum profit rate reveals two things at the same time: one is reflected, obviously enough, by the necessity of prices to change in order to achieve the same profit rate in each sector for the given wage rate; the other, far more important, is expressed by the circumstance that only at zero profit is the *socially optimal* configuration of the economy achieved – this meaning a situation characterised by the largest achievable net product, the latter being wholly paid out in wages.[5]

This is the consequence of the fact that distribution affects the choice of methods of production, and – within the framework of a stationary economy – when the wage rate is fixed at *less* than its maximum, a *lower* net product will emerge with the chosen methods of production. Therefore, with a positive profit rate the share of the net product gained by the producers would be *less* than that lost by the workers. This is crucial for the argument.

In line with this conclusion, Goodwin is then right in saying that 'profit has no function' (*EE*, p. 56) (meaning properly, no *productive* function) and also that when the profit rate is positive then consumption and net product are *sub-optimal*, notwithstanding that a *long-run equilibrium* is established anyway. This situation is judged 'surprising and disturbing' (*EE*, p. 56) for there are no conditions available that could possibly *induce* any alteration of the choices made by the producers as well by the workers.

11. The analysis of a stationary economy gives Goodwin the opportunity to examine the essential content of the labour theory of value and also to clarify the connected concepts of economic 'surplus' and of 'exploitation'.

When the wage rate is at its maximum, the net product is also at its maximum, and therefore the 'surplus' coincides with the net product available for the community. With a positive profit rate, the 'surplus' coincides with gross output minus everything required for production, labour cost included.

In light of the above, Goodwin is in a condition to interpret 'exploitation' in two different senses. There is 'exploitation' because – under the stated condition of a stationary economy, where 'profit has no function' – a redistribution from profits to wages is not harmful for production. There is 'exploitation', however, also because with a positive profit rate the net product is not at its maximum, and therefore prices and methods of production are sub-optimal, albeit in their long-run condition of equilibrium, as we have seen above.

Goodwin's notion of 'optimality' within the framework of a stationary economy seems to have created some uneasiness in one reviewer of *EE*. (Bliss, 1971, p. 626). What Bliss considers quite disturbing, in fact, is Goodwin's attempt to structurally link 'sub-optimality' with a 'positive profit (interest) rate'. The example produced by Bliss in order to confute Goodwin's thesis, however, appears absolutely off the mark, for in presenting 'sub-optimality' by means of an oxymoron ('very poor people live on profit' and 'very rich people live by their labour') and then asking 'Is such an economy sub-optimal if the interest rate is positive?', he seems to reject such a situation only for its being 'absurdly unrealistic' – as if the situation pointed out by Goodwin were of empirical content instead of theoretical.

12. Some of the most enjoyable reading of *EE* are doubtless those pages dedicated to the notion of 'capital'.

In the late 1960's a much debatable controversy on capital theory (and indirectly on the neoclassical paradigm) was at its zenith. The few pages Goodwin devotes to the notion of capital are really worth much more than a voluminous treatise on that topic would be, both for his refined treatment of that still questioned concept as well as for the full maturity and deep knowledge of the entire subject he proves to possess.

The most difficult aspects of the controversy are all dealt with softly and with and gentle criticism – a mode that appears millions of miles distant from the way which instead characterized the controversy in those years.

Goodwin's approach recalls what one of the most brilliant and original Italian writers, Italo Calvino, wrote in his *Six Memos for the Next Millennium* under the chapters entitled respectively 'Lightness' and 'Exactitude':

> Whenever humanity seems condemned to heaviness, I think I should like fly like Perseus into a different space. I don't mean escaping into dreams or into the irrational. I mean that I have to change my approach, look at the world from a different perspective, with a different logic and with fresh methods of cognition and verification. (Calvino, 1988, p. 7)

> Certainly he [Leonardo] thought he could set down much of his science more clearly in drawings than in words. (Calvino, 1988, p. 78)

Those pages seem to have been written a hundred years after the capital controversy took place, with all the benefits the simple passage of time grants in such circumstances, coupled with the wisdom of accumulated experience. But much more than this, they seem to have already filtered and condensed in a nutshell the many positive contributions some great economists have given to the treatment of 'capital', from Ricardo to Sraffa, passing through Wicksell.[6]

One cannot resist in giving but a sample of those pages through the following passage:

> Capital is to be regarded as simply a name for the pervasive temporal structure of all economies. There is no thing or group of things which is capital. Rather, inputs precede output and the pattern or time structure of this precedence varies between goods and processes. This fact is of great importance whenever growth is occurring and hence must enter into any price system or valuation which is used as a basis for decisions. Thus the deeper we probe into the concept of capital the more it tends to vanish into the earlier and the later, the temporal structure. To look for a thing called capital represents the fallacy of misplaced concreteness; there are no things called capital; there is only the dynamical interrelation or ordinary things, goods and labour. (*EE*, p. 67)

13. The last chapter, which introduces dynamic programming, is a fantastically pedagogical chapter.

At first sight, it would seem an almost impossible task to perform within the elementary structure which until that chapter has characterised the entire exposition. Even on this topic, however, Goodwin brilliantly succeeds in giving much more than a simple flavour.

As is well known, Ramsey's pioneering work (Ramsey, 1928) is within the framework of the classical calculus of variations, which presages Optimal Control Theory; and the calculus of variations is a particular example of a multistage decision process of continuous type or, alternatively, of the theory of dynamic programming. The latter originates in the Chicago thesis of Valentine (Valentine, 1937), later through Bellman (Bellman, 1957), among others.

Goodwin acknowledged Ramsey in the Preface of his book, as has been reported at the

beginning of this work. Ramsey's 1928 paper, however, has been practically ignored over the years after its publication, at least until Tinbergen made his own contributions. (Tinbergen, 1956 and 1960) The problems of unemployment and inflation before and after the II World War were far more urgent than those of optimal growth – and this might possibly explain that oblivion. And in spite of the 'resurrection' of Ramsey by the orthodoxy – due to Samuelson, 1965, Koopmans, 1965 and Cass, 1965 – nonetheless optimal growth theory still remained neither fashionable nor popular among the economists' community.

Goodwin, however, always looked far ahead of his own time, and presumably he rightly felt the demographic problem with all its dramatic economic aspects a serious problem for humanity in the years to come and from the global viewpoint – as it is in fact quite evident now, although the problem is still underestimated.

Dynamic programming, in effect, gives the most appropriate framework for dealing with demographic problems, and – in the able hands of Goodwin (who treats dynamic programming in a finite horizon, non-stochastic, framework) – it also provides the chance to treat the problems from a genuinely *political economy* perspective in the true Classical tradition.

For example, the market mechanism turns out to be *in*sufficient for the realization of an entire program over a finite sequence of years. As Goodwin puts it:

> The fact that there is no way of isolating and solving by binary pairs of years explains why no market mechanism can ever be expected even to approximate the optimal solution. Markets operate on current and expected prices but these expected prices turn out to be based, necessarily, on past information. What is required for optimal programming is not simply past performance but a complete specification of the behaviour all the way to the end of the plan period. (*EE*, p. 172)

Demographic problems are really very difficult to state properly and, like many other human problems, be they individual or collective, extremely difficult to manage and solve.

In particular, in the case of demographic problems, there is an inescapable intertwine among the conflicting factors – political, economic and ethical – which makes it hard to consider with deftness the birth and the death rates as either control variables or state variables. In these crude but otherwise incontrovertible words, Goodwin states:

> in most cases ethical beliefs rule out intentional withdrawal of medical service to raise the death rate, even though societies readily tolerate economic policies which have the same effects. (*EE*, p. 180)

The contrast between a dynamic programming-based policy and a policy adopted on the basis of a misleading intuition is established by Goodwin with great clarity by the analysis of the effects of a sudden increase in the wage rate in an under-developed economy. Goodwin supposes that this sudden increase in the wage rate is the direct consequence of an action taken by a coup, whose aim consists of immediately raising the standard of living of the population.

The apparently 'right' decision of the coup, however, turns out to be ultimately self-defeating. In fact, the standard of living does go up at first, but as a consequence of this the rate of labour demand goes down (implying that the growth rate goes down too) and the rate of labour supply goes up. In both ways, therefore, the employment rate goes down and so ultimately does the standard of living. The decision of the coup, though initially popular, has been transformed afterwards into a quite myopic decision. An alternative and more appropriate policy would have been that of taking into account the entire time profile of the program, and letting the population bear an initially *lower* wage rate and a lower standard of living as a consequence. After a while, the unemployment would have been eliminated, and in this way a higher standard of living would have been achieved and maintained.

In another model, a strictly Malthusian (and perhaps, even more properly, early Wicksellian) birth rate control is finally shown to possess, in a dynamic program for the population, a clear

superiority over a policy of accumulation aimed at increasing equipment and therefore jobs for the available work force.

The population or labour force can be changed by social and political decisions. The point, however, is to insert the corresponding policies in a coherent *optimal sequence* of decisions.

A few years later, Goodwin came to discuss the relation between Population and Resources from a global point of view, with the background of the Malthusian-Wicksellian 'philosophy' once again.[7]

On that occasion he wrote:

> What is quite certain is that our civilization will depend increasingly on a technology of an ever more complex and all-embracing sort. This looks like meaning an ever diminishing amount of both freedom and democracy. (Goodwin, 1978, p. 198)

This perhaps sounds like one of the most bitter, and yet severe, warnings he left to all of us as human citizens of the global community.

Footnotes

[1] The proof was given by Andrew Wiles and Richard Taylor, 1995. In classical mechanics, to take another example, the *two-body problem*, i.e. the problem of computing the mutual gravitational attraction of two masses, can be completely solved. By contrast, the corresponding *three-body problem*, although *theoretically* solvable, has a solution implying series which are unusable 'either as a means of obtaining numerical information or as a basis for numerical computation', Birkhoff, 1920, p. 53. Cfr. also Barrow-Green, 1997, p. 191.

[2] See the interesting brief summary report of a conversation between Goodwin and Velupillai on Walras' Theory of *tâtonnement* in Velupillai, 1998, p. 17, fn 32.

[3] It should be noted that the notion given by some Sraffians (Garegnani, 1970, p. 418; Pasinetti, 1977, pp. 78-79 and p. 97) reproduces the Hawkins-Simon condition. For a discussion on viability may I refer to Chiodi, 1998, and 2010.

[4] A comprehensive and detailed analysis of Goodwin's main contributions on this topic can be found in Velupillai, 1982 and 1990.

[5] This is also the thesis which was maintained by Schumpeter, as reported by Goodwin himself. (*EE*, p. 56). In a personal conversation, Goodwin told the writer that Schumpeter had some discussions with Böhm-Bawerk on this thesis.

[6] Goodwin was an admirer of Wicksell. Cf. Velupillai, 1998, p. 1442. He wrote a lovely note for the flap of my book on Wicksell. (Chiodi, 1991)

[7] The paper was originally presented at "The Arne Ryde Symposium on the Theoretical Contributions of Knut Wicksell", held at Frostavellen, Sweden, in the Fall of 1977. I had the great pleasure of being there.

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