A Crooked Path along The Gravel Walks

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“River gravel. In the beginning, that.
High summer, and the angler’s motorbike
Deep in roadside flowers, like a fallen knight
Whose ghost we’d lately questioned: ‘Any luck?’
……
But the actual washed stuff kept you slow and steady
As you went stooping with your barrow full
……
So walk on air against your better judgement
Establishing yourself somewhere in between
Those solid batches mixed with grey cement
And a tune called ‘The Gravel Walks’ that conjures green.’
In the half-century of my life as an Economist, the works, correspondence and lives of four Universalists has dominated my own interests – but also four others whose works, over the years, came gradually to influence me: Gunnar Myrdal, the Universal Social Scientist (Velupillai, 1990), Maynard Keynes, the Universal Man¹, Paul Samuelson, the Universal Economist (Velupillai, 2018), and Herbert Simon, the Universal Behavioural Scientist (Velupillai, 2017).

But the other four, those to whom I am equally indebted, for their ideas and methodologies, were no less Universal in the doctrines they advocated: Richard Goodwin, Piero Sraffa, Luitzen Brouwer and Alan Turing².

They all uphold the tradition of a Leonardo da Vinci, in the sense of being latter-day renaissance men – the only difference being that I had the privilege of knowing all of them, personally, except Brouwer and Turing. There is, therefore, no excuse whatsoever for me not to be humble, be measured and careful in my claims and be unreservedly modest in any analytical framework I think I have constructed – especially if it is to be consistent with the impossible task of synthesizing the diverse strands of their methodological frameworks.

So, The Gravel Walks have dominated my intellectual pursuits, in my search for a synthesis of the apparent diverse frameworks, these eight Universalists have fashioned, for analytical understanding of dynamic mathematical behavioural economics, in a constructive and computable mode.

On 12th September, 1996, the late Paul Samuelson wrote me as follows:

“You must fill me in on exactly how Goodwin contributed toward solving Hilbert’s 16th problem.”

² They, too, have fashioned ‘Universalities’: Universal Macroeconomic Dynamics (Goodwin), Universal Classical Economic Value Theory (Sraffa), Universal Intuitionistic Constructive Mathematics (Brouwer) and Universal Machine Computation (Turing). The perceptive reader may have noticed that six, of the eight, are portrayed in the ASSRU Logo. The time will come when the hexagon of people will become an octagon! By the way, there is such a thing as the Universal Turing Machine and, in my opinion, Sraffa’s construction of the Standard Commodity, the constructive proof of its uniqueness and the Standard Ratio are a contribution to a Universal (Classical) Theory of Value. I also believe that the ‘forced van der Pol’ oscillator has Universal dynamic and algorithmic properties. The Brouwer-Heyting-Kolmogorov (BHK) methods of proof are Universal.
Samuelson was referring to my claim that Le Corbeiller’s student (Goodwin, too, was a ‘student’ of this great applied mathematician), Rui Pacheco de Figueiredo, in his Harvard applied mathematics doctoral dissertation, had used what he (and Le Corbeiller) had referred to as the ‘Goodwin Oscillator’ (one that, contrary to conventional wisdom, generated closed paths, that were stable) which was endowed with only one-bend³ (unlike the van der Pol and similar oscillators). de Figueiredo had used this one-sided oscillator, as Goodwin referred to it, to solve, for the Liénard (Nonlinear) Equation, and give an acceptable⁴ answer – even if non-constructive in its proof - to the second part of Hilbert’s 16th Problem.

The point was that a geometric construction that Goodwin had developed, for entirely economic reasons, had been used to resolve – even if only partly – a celebrated ‘problem’. I asked myself, whether something similar could be said, or demonstrated, on the basis of one of many of Simon’s innovative constructions. I confined myself to four of Simon’s conceptual constructions: Heuristics, Information Processing Systems, Bounded Rationality and Satisficing; however, bounded rationality and satisficing underpinned the construction, definition and activities of the Information Processing System.

Hence, in Herbert Simon’s world, Human Problem Solving was implemented by Information Processing Systems, utilizing Heuristic (algorithmic) Processes.

To try to find an application of the Simon conceptual constructions, in the attempt to solve one or another of the known famous problems, I ‘scoured’ the literature and decided (sic!) to look at the following list:

- **Hilbert’s Problems** – specifically, the 10th Problem (cf., Hilbert, 1900; 1902); Davis, 1973; Matiyasevich, 1993);
- The Clay Mathematical Institute’s Millennium Problems – specifically that which was on the question of whether $P \neq NP$ (J. Carlson, et.al., editors, 2006, see, in particular, the chapter by Stephen Cook);
- The Four-Color Problem and the Appel-Haken Computer Aided Proof of Guthrie’s conjecture, which became a theorem, and the search for its mathematical proof as the Solution (Appel & Haken, 1976; Saaty & Kainen, 1977; 1986);

³ Goodwin (1950), developed the idea entirely geometrically (by ‘free-hand’ drawing of possible macrodynamics), when reviewing Hicks (1950), essentially showing that either the ‘ceiling’ or the ‘floor’ was sufficient to generate an endogenous closed path in a two-dimensional dynamical system.

⁴ I chose the word ‘acceptable’ guardedly, mainly because of the controversy surrounding the claims, in this regard, by a ‘young’ Uppsala (applied) mathematician.
• Smale’s *Mathematical Problems*\(^5\) for the 21st Century (Smale, 1998); especially *Problems 3 & 18, Does P = NP? & the Limits of Intelligence*, respectively.

For a very long time I had wondered how one would approach the unsolvability of *Diophantine equations* with the conceptual apparatus that Simon had provided – particularly whether an *Information Processing System* could, using *Heuristics* (in the way Newell & Simon had done, in their monumental text of 1972, *Human Problem Solving*), find a way to approach the issue in the way Davis, Putnam, Julia Robinson and, finally, Matiyasevich, had done.

Eventually, I came to the ‘melancholy’ conclusion that any method of solution of *exponential Diophantine equations* had to invoke a *formalized* process for the necessary proof and, therefore, I had to find an equivalence between *Heuristics* and a form of the *Church-Turing Thesis*. At this point, the *decidability* issue seemed tied to *Turing Machine Computability*, predicated upon the (implicit) assumption of a form of the Church-Turing Thesis - and I was convinced that Simon’s (and the constructivists – whether Brouwerian *Intuitionists* or Bishop-style mathematicians) disinterest in proving – using, in particular, *tertium non datur* - the unsolvability of Hilbert’s Tenth Problem was entirely justified.

This was similar to the way I interpreted the *disinterest* – bordering on the proverbial ‘deafening silence’ - shown by Simon (and Newell) to the P ≠ NP question\(^6\), which was an important problem in both Smale’s list of *Mathematical Problems for the Next Century* and the Clay Mathematical Institute’s *Millennium Problems*.

Incidentally, Alan Turing used *Turing Machine Computability* - predicated upon a variant of the Church-Turing Thesis - to make sense of the *Trefoil Knot* and introduce the unsolvability of the *Word problem*; ‘unsolvability’ requires a definition of ‘solvability’ and, hence, his (last past published paper was titled) *Solvable and Unsolvable Problems* (Turing, 1954). I was wrong to think that Turing’s machine computability was equivalent to Simon’s *Human Problem Solving by Information Processing Systems*. The former was a question of *Machine

\(^5\) *The Riemann Hypothesis* and what was the second-part of Hilbert's 16th Problem (neither still completely solved) – are included also Smale’s list as the 1st & the 13th Problem, respectively; the P ≠ NP question, the 5th in the Millennium Problem list is the 3rd in Smale’s Mathematical Problems for the Next Century.

\(^6\) After all, NP stands for non-deterministic, polynomial time (Turing) computability! See also Simon’s feeling of *ennui*, and the context in which it arose.
Computable; the latter was predicated upon Heuristics (of Search Processes, assuming Gödel’s Completeness Theorem for Propositional Logic), or an intuitive understanding of Mental constructions of processes that solved problems – in the sense of Brouwer (and BHK).

That leaves the Computer Aided Proof of the four-colour theorem – Guthrie’s Conjecture – and Problem 18: Limits of Intelligence, in Smale’s list, both of which, I think, could be approached via the conceptual tools constructed by Simon, i.e., use of Heuristics by Information Processing Systems.

It is serendipitous to remember that it is almost exactly 60 years since the Dartmouth Conference, gave birth – by, among others, Herbert Simon – to the concept of Artificial Intelligence (and almost twenty-years since Smale, 1998). However, priority – as handsomely acknowledged by Simon (1996) – in this respect, albeit with respect to Machine Intelligence, must go to Turing (1950)\(^8\). Although Penrose (1989, 1994) tries ‘to show some limitations of artificial intelligence’ (Smale, ibid, p. 13), I subscribe to the cogent – computability-based – refutations of the Penrose-thesis\(^9\) given by Davis (1996) and Putnam (1995), respectively.

Smale (ibid, p. 13; italics in the original), significantly, states Problem 18; Limits of Intelligence, as:

What are the limits of intelligence, both artificial and human?

\(^7\)My opinion – not something I can unambiguously attribute to Herbert Simon.

\(^8\)In interpreting Machine Intelligence as Mechanical Intelligence – the title of one of the volumes of the Collected Works of A. M. Turing (1992, edited by D. C. Ince) – I can, at last, find a way to ‘equate’ Turing’s Machine-based solvability and Simon’s notion of Human Problem Solving (also emphasised by Simon, 1996). In this connection, see also Gandy (1996).

\(^9\)It must be remembered that Penrose believed – and reasoned on this belief – that the mind resided in the brain and, hence, ‘is concerned with the brain’ (Gandy, op.cit, p. 135). Simon (1996, p.81), is explicit in stating:

“I speak of ‘mind’ and not ‘brain’.”

And as Gandy points out (ibid, p. 136; italics added):

“[P]enrose asserts forcefully that consciousness is essential to rational thought; I do not quite understand what he means by consciousness.”

Gandy’s modesty prevents him from asserting that ‘rational thought’ has nothing to do with ‘consciousness’.
Remark 1:
I do not believe either Turing, or Simon (or Brouwer, for that matter) subscribe to the view that there were limits to intelligence, in any determinable sense.

Remark 2:
Both Turing and Simon defined, pro tempore, notions of intelligence (as a process), the former subject to a form of the Church-Turing Thesis, and the latter in terms of (formally undefined) Heuristic processes; in Brouwer’s case, it was subsumed in his notion and concept of the creative mathematician.

Hence,

Conjecture 1:
The problem of ‘the limits of intelligence’ is (algorithmically) undecidable.

Remark 3:
The proof is ‘easy’, provided one can ‘accept’, unambiguously, any of the (formal) definitions of intelligence, given by Turing or Simon.

Finally, there is the computer aided proof of Appel & Haken (1976), of Guthrie’s Conjecture of the planar four-colourability of a map (Saaty & Kainen, 1986). I think this computer aided proof is reproducible, using Simon’s notion of Heuristics, implemented by Information Processing Systems – and would be fully endorsed by Herbert Simon. As Saaty & Kainen observe (ibid, p.95; italics added):

“Appel and Haken began with an initial discharging algorithm and successively modified it. This [modification] involved considering the possible failure cases of each algorithm, which could occur because of the ‘overcharging’ of some vertex; i.e., transferring to a vertex with negative charge too much positive charge. The enumeration of cases was done by a [digital] computer, changes were made, and then the new algorithm was again examined. In other words, the computer was used via man-machine interaction as a ‘scratch pad’ in order to find an appropriate discharging procedure.

The next step in the proof, checking configurations for reducibility, also employed the computer. … Human computation [alone] is simply too slow.

[.]ny proof of the four-color theorem …. must be extremely complex, requiring computer assistance.”

Reading heuristics for algorithm – Appel and Haken do not invoke (in fact, cannot) any form of the Church-Turing Thesis, and cannot implement any form of the BHK process in their automated proof - and the modification is to be (partly automated) by information processing systems, it is possible to show that this is exactly how Simon (and Newell) envisaged Human Problem Solving.
Remark 4:
Appel & Haken also invoke, felicitously, *Heuristic notions*, but in limiting the possible *probabilistic* alternatives in the proof process, thus *reducing* the cases to be considered; Simon’s *Heuristics* are implemented in *constrained deterministic spaces*, by *information processing systems*.

I can, therefore, conclude with a conjecture:

**Conjecture 2:**
The four-colour theorem *can* be proved *mathematically* – but with the aid of a digital computer.

Remark 5:
The proof of Conjecture 2, I surmise, would be within an (constrained deterministic) iterative space, subject to Gödel’s Completeness Theorem for *Propositional Logic*, implementing Simon-type *Heuristics* by *Information Processing Agents*.

Remark 6:
The ‘constraints’ are determined by the *bounded rationality* of *Information Processing Systems*, solving problems by *satisficing*.

The *Crooked Path* along *The Gravel Walks* was a long journey, even if rewarding for the many illuminating, intellectual and humane insights. It remains unfinished, *incompletable* and is suffused with ambiguous unpredictabilities, with forks replete – often - with undecidable disjunctions. This is only an interim report, full of compromises and promises.

I am reminded of Hannah Arendt’s (1998; 1958) poignant words, in *The Human Condition* (p. 232; italics added):

“The central concept of the two entirely new sciences of the modern age, *natural science* no less than *historical*, is *the concept of process*, and the actual human experience underlying it is action. Only because we are capable of acting, of *starting processes* of our own, can we conceive of both nature and history as *systems of processes*.”

It is no exaggeration to think that Herbert Simon is in that great tradition of thought, and action, which begins with *Vico* (*history as a process*), continues with *Newton natural Science as a process*) and *Brouwer* (*mathematics as a mental process*), then begins a new ‘modern age’ with *Turing* who made *computation a process*. Herbert Simon fashioned behavioural sciences as a process, in this noble, centuries-old, tradition. It is, now, a *Vico-Newton-Brouwer-Turing-Simon* tradition of a *new Modern Age*. 
References

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