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REMEMBERING CLOWER[^]

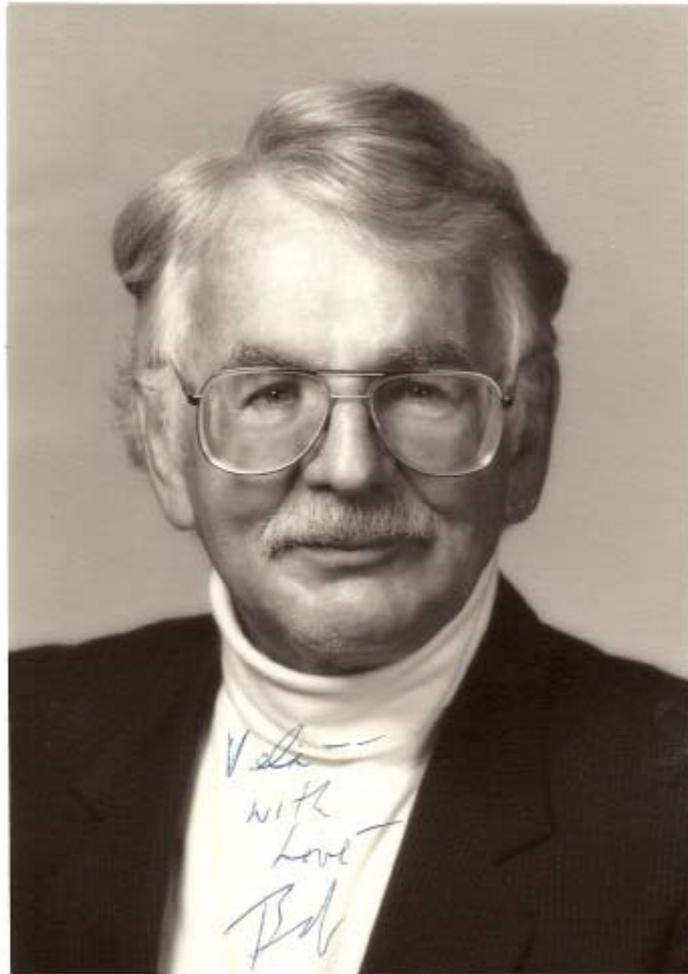
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“The upshot of all this is that everything being done these days by the general equilibrium idiots with "money" is just silly. Until they give up the idea of general equilibrium *à la* Walras and deal with a conception of an economy that actually has a money commodity explicitly built into it, they will continue to talk nothing but nonsense.”

Letter from Clower to the author, 27 November, 2006

[^] An earlier, more concise, version was published in the **Economic and Political Weekly**, Vol. XLVI, #24, June, 11-17, 2011. I am indebted to the Editor of EPW for permission to incorporate some of the material in the earlier version in this paper.



Robert Wayne Clower
13 February, 1926 – 2 May 2011

§1. A Preamble

“My ‘doubts about orthodoxy’ occurred long before I knew any economics. What others may see as an intellectual development, I knew to be a personality trait.”

Clower, 1984, p. 259.

With the death of Robert Clower, on 2 May, 2011, the group of outstanding Monetary Macroeconomists² who moulded the subject in terms fashioned by Knut Wicksell, Irving Fisher, Maynard Keynes and Dennis Robertson, has lost its last, enduring, exponent. This set of giants, whether we agreed with their notions and ideologies, their theories or their methods (or lack of method), comprised - apart from Robert Clower - Paul Samuelson, Milton Friedman, Don Patinkin and James Tobin – an all American group. This was in contrast to the pre-War pioneers of Monetary Macroeconomics, all of whom, with the exception of Irving Fisher, were Europeans: Wicksell, Cassel, Keynes, Robertson, Lindahl, Myrdal, Ohlin and Hayek. John Hicks straddled both eras.

The only explicit statement the great logician Kurt Gödel is ever known to have made about economics was characteristically prescient. Anticipating, by many months, the publication of Keynes’ *General Theory*, Gödel, in commenting on a celebrated paper by Abraham Wald which, itself, was the fountainhead of modern mathematical general equilibrium economics, pointed out a central tenet of Keynesian macroeconomics: the need to include *income* as an argument in individual demand functions. The famous Clowerian *dual decision hypothesis*, hatched in Monrovia, the capital of Liberia, and produced at the Abbey in Royaumont, France at the *International Economic Association* conference of 1962³, was devised with the express intention of ‘getting income into demand functions’ and, thus, reorient the research agenda of macroeconomic theory away from the fallacies and *ad hoc*eries of the neoclassical synthesis toward a framework where money, dynamics and disequilibria played essential roles with solid microeconomic foundations. It was the

² I must, at the outset, confess that I have never subscribed to the bracketing together of Leijonhufvud’s name with Clower in an understanding of the latter’s vastly more profound, analytically deeper and doctrine historically wider – encompassing not just the history of economic thought but also mathematics and philosophy of science – scholarship in the underpinning of monetary macroeconomics *after* Keynes. The dynamic, disequilibrium basis of the monetary macroeconomics Clower sought to develop, as a *General Process Analysis* (Clower, 1984, Part IV) has nothing whatsoever to do with what Leijonhufvud has tried to popularise as *Adaptive Economic Dynamics*, and, in any case, the latter is not envisaged in terms of monetary disequilibrium dynamics, especially in a macroeconomic mode.

³ Ironically, it was first published in *German*, in 1963, in the *Swiss Journal*, *Schweizerische Zeitschrift für Volkswirtschaft und Statistik* (see, Clower, 1984, footnote on p. 34)!

beginning of a lifelong search for consistency between macro and micro economic theories which, Clower thought, could be found only if the economic theorist succeeded in constructing successful *thought experiments (Gedankenexperiment)* to tame disequilibrium economic processes in monetary economies. This search for consistency was, surely, the only bond that linked the unlikely personalities of Gödel and Clower.

I first met Bob Clower at the History of Economics Society meetings in Boston, in June, 1987. The very first words I exchanged with him had to do with a particular way I had introduced my subject, which was on *The Political Arithmetics of the Stockholm School*. As a way of justifying, even if only pro tempore, the stance and interpretations of the textual material from the fifty-year period, 1889-1939, essentially from David Davidson to Erik Lindahl, of the Swedish contribution to macroeconomics⁴, I said: '*Stories need not be true to be believed; they only have to be plausible and interesting.*' This was a slightly different justification of the philosophy of an 'as if' working hypothesis (different from the instrumentalist way popularised by Friedman (1953)) in the domain of the history of economic thought, perhaps along the lines elegantly described by Vaihinger (1925).

Clower obviously approved this stance and came up to me immediately after my talk and we had a most pleasant and, for me, a very rewarding conversation about *methodology, mathematics, monetary theory and macroeconomics* – the four domains that defined my professional interactions with Clower for the next (almost) twenty-five years. His knowledge and expertise on methodology – not just economic methodology – was unusually vast and deep. He was exceptionally well informed on Charles Sanders Peirce and Norwood Russell Hanson, and over the years I benefitted greatly from this knowledge, especially when I was struggling with the notion of

⁴ Leijonhufvud (1981), in his unfortunately 'influential' *Wicksell Connection*, popularised the diametrically opposite view (perhaps thereby, inadvertently, influencing Woodford (2005; p. xiv) to propagate the same vision):

"The central concepts of Wicksell's analytical apparatus are, of course, the *market rate* and the *natural rate* of interest. The terms are names for two values of the same variable." (p. 155; italics in the original)

Nowhere in the Wicksellian classics, nor in any of the monetary macroeconomic writings of Lindahl or Myrdal have I found any evidence to substantiate this assertion. Only someone with a slippery mastery of the deep conundrums of capital theory could make the above kind of remark.

retroduction, the word used by these two American giants for the more traditional word *abduction*.

His pioneering contributions to monetary macroeconomics – both pure monetary theory and macroeconomics in its dynamic variants – were legendary and I was fortunate to belong to the generation that was brought up on his writings. They fashioned and moulded the emerging microfoundations revolution in macroeconomics, long before Phelps and others began trying to claim that mantle and domain.

But it was his extraordinary knowledge of mathematics, particularly of dynamical systems theory, that – for me, in any case, that was most impressive. It was only later that I was able to realise that part of the reason for this particular expertise was his lifelong friendship and collaboration with Donald Bushaw, with whom he wrote one of the most important contributions to macroeconomic dynamics in the early 1950s (Bushaw & Clower, 1954) – sadly submerged in the Arrow-Debreu avalanche, and ironically their original classic (Arrow & Debreu, 1954) was published in the same issue of *Econometrica* in which the Bushaw-Clower classic appeared.

The personal friendship which became intensive, was built upon these professional foundations, but took on a completely different and independent life, also warmly intermediated by his charming and engaging wife, Georgene.

§2. The Fifteen Clowerian Codes⁵

“It has been suggested that contemporary microtheory deals with ‘pinhead’ societies in which even ‘large, square economies’ (with a continuum of commodities and agents) can be ‘placed on the head of a pin. Of course, mathematical concepts can be ‘placed’ metaphorically as easily on a debating point as on a pin. But before this

⁵ I first used the concept of ‘codes’ in macroeconomic contexts, derived from ‘codifying’, to encapsulate and summarise *Theories of the Business Cycle: From Frisch to Lucas* in my UCLA Lectures of 1987. Clower’s use of the phrase *Neowalrasian Code* (see §4, below) derives from Howitt (1996). My use of ‘codifying’ and ‘codes’ was juxtaposed with the way I began to use the word ‘closure’, a few years before 1987, when my research program on computable economics got underway, around 1983. It was only in very recent months, during interchanges with Lance Taylor, I came to realise that Lance had used the word ‘closure’, albeit in a slightly different sense, already from the late 1970s (cf. Taylor & Lysy, 1979). My use of the word ‘closure’, closely allied to the way I used ‘codes’ – similar in vein to the way it was used by Howitt (op.cit) – and ‘codifying’, was in the mathematical sense of the ‘closure of a theory’.

footnote becomes pointless, let me make its point: my ‘definitions’ of contiguity and connectedness are meant to be suggestive, not formal.”
Clower (1996), p. 55.

Bob Clower, almost single-handedly, initiated simultaneous moves away from the complacency and the twin orthodoxies of the *neoclassical synthesis* that had come to dominate *macroeconomic* thinking on the one hand and, on the other, a *general equilibrium theory* that had come to be viewed as providing a rigorous foundation for microeconomic theory. Clower’s celebrated two *thought experiments* – the *dual decision hypothesis* and the *dichotomised budget constraint* - were constructed, respectively, to expose the lack of any Keynesian content in the neoclassical synthesis and to highlight the role of money in exchange processes and, hence, the vacuity of a general equilibrium theory that claimed to model exchange economies faithfully.

These two almost synchronised moves - away from the orthodoxies of monetary macroeconomics and mathematical microeconomics - were *codified* in terms of what I have come to encapsulate as the *15 Clowerian Codes*:

- i. Stock-Flow Monetary Dynamics
- ii. The Patinkin Controversy
- iii. Hahn’s Problem*
- iv. Say’s Law (Lange)
- v. Walras’ Law (Lange)
- vi. The Keynesian Perplex
- vii. The Neowalrasian Code
- viii. The Neoclassical Synthesis
- ix. The Dual-Decision Hypothesis/Dichotomised Budget Constraint
- x. The Keynesian Cross
- xi. Thought Experiments
- xii. Induction
- xiii. Axiomatics
- xiv. General Process Analysis
- xv. Non-Convex Analysis and Dynamics (‘Money, Credit and the Timing of Transactions’)

I have listed them in the *approximate* sequential order in which they were codified by Clower, in his fascinating, constructively critical, intellectual journey towards a disequilibrium, non-conves, dynamic monetary macroeconomics, underpinned by a rigorous, yet empirically disciplined microeconomics. However, a discerning reader would, in any case, be able to detect a pattern in the sequence, transcending the ‘timeline’.

For example, (xi) – (xiii) – thought experiments (Clower, 1984, p 82), induction (Clower, 1994) and axiomatics (Clower, 1995) – form a methodological triptych. However, the fruitful use of thought experiments, in theorising counterfactually, permeate Clower’s contributions almost without exception. I detect the influence of Charles Sanders Peirce (1966; particularly Volume VII, Bk. II) and Norwood Russell Hanson (1969) in these forceful methodological reflections and admonitions for and to economists.

The first and the last two form a unified couple, providing mathematical economic foundations for a general process analysis, where disequilibrium economic dynamics is encapsulated in an intrinsic stock-flow dynamics. They emerged, almost fully formed for use for the rest of his intellectual life, in that *annus mirabilis* for Clower⁶, 1954, in Bushaw-Clower (1954), Clower (1954a) and Clower (1954b).

The formalism of a general stock-flow dynamics was achieved by Bushaw and Clower in terms of Integro-Differential equations; however, for purely economic reasons, it may have been far more intuitively useful to have formalised stock-flow dynamics in terms of differential-difference equations. Had they done the latter, their classic may have fared better as an intermediate precursor of the ‘time-to-build-tradition in business cycle theory (cf. Dharmaraj & Velupillai, 2011, for a fairly comprehensive survey of this tradition).

⁶ All of the papers that came to be published in 1954 had been conceived and written in 1952/3, i.e., when he was not quite 27 years old. This fact puts in perspective Clower’s mature reflection (Clower, 1984, pp. 263/4):

“Economics is less obviously a young man’s game than mathematics, but I doubt that many economists have had a really new idea after the age of thirty-five. I must confess that, one way or another, everything I have done in the second half of my life reminds me (at least retrospectively) of something I did or thought about earlier. This is especially true of my papers on money, which are full of rearranged ideas, some drawn from earlier published work on unrelated subjects, others from ruminations that earlier came to nothing .”

It may be apposite to mention that the Bushaw-Clower classic was the first article in economics to mention clearly, and utilise imaginatively, Lyapunov's stability criteria, at least in the English language economic literature⁷. Moreover, so far as I am aware, this was the very first paper in economics, in any language, to have referred to the important Russian contributions to an analysis of the problems posed by linearizations of nonlinear differential equations and the difficulties of inferring, from the stability of the linearised system, anything about the dynamics of the more general nonlinear system from which it was derived. Bushaw and Clower refer, explicitly, to the relevant sections in Russian classic by Nemytski and Stepanov (cf. Clower & Bushaw, footnote 8, p. 337). Obviously, they were familiar with this literature in view of the important role played by Bushaw in the editing of the translation of the first edition of the Nemytski-Stepanov classic⁸.

In Monetary Macroeconomics Clower's lifelong preoccupations were centred on untangling the confusions wrought by the *Neoclassical Synthesis* as a repository of Keynesian Macroeconomics, first by Hicks and his unfortunate IS-LM mythologies⁹ and, later by Lange¹⁰ (1942) and Patinkin¹¹ (1956, 1965), with their *neo-Walrasian* phantoms, through which prisms *the classical dichotomy*, *Say's Law* and *Walras's*

⁷ Weintraub (1991) has explored and reported the Japanese contributions by Yasui and Morishima, who both seem to have used Lyapunov's methods in the late 1940s. I am not entirely sure their memories, faithfully reported by Weintraub, are a reliable guide to the exact or accurate sources on which they relied for the Lyapunov theory. This is because, as a result of Perron's work on generalising some of the Lyapunov results, there were fundamental and very early contributions by eminent Japanese mathematicians to the stability theory of differential equations which easily shows full familiarity with the 'Western' literature (cf., for example Fukuhara & Nagumo, 1930).

⁸ Solomon Lefshetz, who was instrumental in arranging for this translation, acknowledged in the *Preface to the English Language Edition of Nemytskii & Stepanov* (1964), as follows (p.v):

"This translation [of the English language edition of the Nemickii-Stepanov treatise] was edited by Donald Bushaw and John McCarthy, at the time graduate students at Princeton University."

That very little of this has received adequate recognition in the economics literature – instead attributing various priorities to Arrow, Block & Hurwicz, Allais, etc., may well be due to the unfortunate fact the Bushaw-Clower classic appeared in the same issue of the **Econometrica** as the one in which the (infamous!) Arrow-Debreu classic appeared!

⁹ It may be useful to point out that the Hicks classic (Hicks, 1937) labelled the two intersecting lines in the r-Y space as SI & LL. Any moderate cynic, reading from left to right, could not but have wondered why the whole thing was SI-LL Y!

¹⁰ Clower considered the pernicious influence of Lange's chapter in the **Schulz Festschrift** almost unforgivably irreparable, although he made valiant attempts to set right the logic, the analysis and the doctrine historical background to both Say's Law and Walras' Law – even till the very end of his life (cf., Clower, 2004).

¹¹ There is no better indictment of Patinkin by Clower than in his disdainful dismissal of the former's (in)famous opening line in Patinkin (1965, p.xxiii), as 'nonsense': 'Money buys goods and goods do not buy money.' Instead, Clower suggested the 'aphorism' (Clower, 1984, p.86):

"Money buys goods and goods buy money; but goods do not buy goods."

Law came to dominate monetary discussions. To these was added the so-called *Hahn Problem* of the difficulties in proving the *existence* of an equilibrium in a monetary economy. The core monetary macroeconomic contribution by Clower are encapsulated, thus, in the *codes* I have called the *Patinkin Controversy*, the *Hahn Problem*, *Say's Law*, *Walras' Law*, the *Keynesian Perplex* and the *Dual-Decision Hypothesis*.

I believe – at least if I am to go by what I learned from personal conversations with Clower – he came round to the view that the *Hahn Problem* was a classic red herring in the following sense. An essential monetary economy is a disequilibrium dynamic process. Therefore, any attempt to show the difficulties of proving the existence of equilibrium in a static ‘monetary’ economy, using classic non-constructive methods was a pointless, non-empirically underpinned, exercise. I wholeheartedly agree. Mercifully, the Hahn Problem has been forgotten in the monetary macroeconomics literature at the frontiers- but, unfortunately for all the reasons that Clower abhorred.

Then, there is the fudge – both analytical and pedagogical – that is the *Keynesian Cross* has been dissected with merciless clarity and precision by Clower (1996), who then goes on to derive and place, with impeccable faithfulness to Keynes (1936, chapter 3), *Keynes on the Cross!*

Finally, the *Neowalrasian Code* has been the *bête noir* of Clower's focused intellectual assault on orthodoxy's complacency in the face of empirical anomalies, historical irrelevancies, institutional vacuities and behavioural absurdities of orthodox mathematical economics.

§3. Outlining an Affectionate Curmudgeon's Life

“I may be a bit of a curmudgeon, but I believe 95 percent of all economists are incompetent, and the rest are anxious to get into public policy.”

Robert W. Clower

In one of those compelling coincidences of life and its mysteries, Robert Wayne Clower was born in Pullman, in the state of Washington, on 13 February, 1926, even as the death was being announced of another great economic theorist, Francis Ysidoro Edgeworth, on that very day. Wicksell, too, died that same year, a few months later.

His father, Fay Walter, was from Morrisonville in Illinois and was a Professor of Economics at Washington State University, in Pullman. His mother, Mary Valentine, from Chanute, Kansas, was ‘wise and lovable’ whose advice, ‘*not to go looking for arguments*’, the son ignored for most of his life. Bob Clower once told me that one of his distant cousins had traced the Clower’s back to about 1745 in Scotland. The first Clowers in the ‘New World’ were found in Charleston, South Carolina around 1750.

Clower graduated from Pullman High School in 1943 after a short stint as a radio repairman in Seattle. He claimed to have won no prizes of distinction at school except to have narrowly avoided occupying last place in the academic honours list at school, 52nd out of 54! After army service for 31 months, from 1943 to 1946, in Wiesbaden, Germany – during which he also managed to get married to a Scottish Lady from Aberdeen – Clower returned to enter Washington State University as a Freshman in the summer session of 1946. Although Clower started his undergraduate studies with the intention of pursuing a career in law, he ended up by graduating, in 1948, with the highest honours and obtaining a BA in economics and, in the following year, also an MA in economics.

Part of these alleyways into economics was inspired by a perceptive Father – “who had met Maynard Keynes at a University of Chicago seminar in 1931 and thought him ‘fascinating’” – who gave the undisciplined son a copy of *The Economic Consequences of the Peace* soon after the return from Wiesbaden and just before entering University, in early 1946, just before Keynes’s untimely death in April of that year. The result was a lifelong passion and interest in Keynes in every sense imaginable and, more importantly, a reading of the *General Theory* even *before* starting formal University education in economics.

A Rhodes Scholarship took him to Oxford to do a Ph.D in economics under John Hicks. Clower initially proposed a thesis on ‘*Theories of Capital Accumulation*’ but changed it to add the phrase ‘*With special reference to Their Ability to Explain the Experience of the United States and Great Britain Since 1869*’, on the advice of Hicks who informed him that to obtain an Oxford doctorate in economics one would have to ‘exhibit skill at handling facts along with theory’. The completed thesis, on

submission in May of 1952, was judged – in Clower’s own brutally honest opinion, ‘rightly’ – to be ‘not in a form fit for publication’ and was failed. He was offered the consolation prize of a B.Litt and a quarter century later, in 1978, Oxford bestowed on him a D.Litt and his Oxford odyssey was crowned, in December of the same year, when he was elected to an Honorary Fellowship at his alma mater, of sorts, Brasenose College.

For a decade and a half he shepherded, as Managing Editor, two of the most influential Journals in the world of academic economics. From 1973 to 1980 he edited the *Western Economic Journal*, which subsequently became *Economic Inquiry*; and the *American Economic Review*, the premier Journal of the profession, from 1981 to 1985. When I asked him, not long after he resigned from the latter, prestigious position, why he did not continue in the post, at least so that he could guide the direction of research, even in some small way, in wise and interesting directions, his answer was pungent and direct: ‘the submissions were pure paper, nothing but paper and dull, dull, dull!’ I could not argue, and did not do so.

Robert Clower unleashed a ferocious and sustained criticism of what he felt was the mechanical framework and duplicity of the neoclassical synthesis and its emasculated portrait of Keynesian economics as a special case of the classical system. In his unfortunately unpublished ‘**Perugia Lectures**’ of the early 70s he described a conversation with Paul Samuelson in the late 1960s. In that conversation Samuelson ‘confesses’ that the phrase *neoclassical synthesis*, first used in the third edition of his famous textbook, was coined simply to placate the ‘McCarthyites’ who were ‘on his back’ for being a Keynesian. Clower’s incredulity, described vividly in those fine ‘**Perugia Lectures**’, that a whole generation of macroeconomic education had been based on an empty shell, and his anger at the callousness of the ‘eastern establishment’, was evident in the written description of that episode.

Clower travelled widely – early in his career to Lahore as a Visiting Professor at the University of the Punjab in 1954-56; to direct the Economic Survey of Liberia in 1961-2; as a Visiting Professor at Makerere College in Uganda, in the Summer of 1965. These visits to South Asia and West and East Africa, he once said, gave him ‘a feel for theoretical treatment of fact and factual scrutiny of theory’.

Then there were invitations and visits to all the usual prestigious places in the western academic world as a Visiting Professor: Cambridge, Oxford, Stockholm, Vienna, Siena, Trento, Western Ontario etc. He even almost decided to move to the UK on a permanent basis in the late 60s when, between his main permanent, long-term, sojourns at Northwestern University (1958-64, Chairman for the whole period) and UCLA (1971-86), he spent a couple of years as the Dean of the School of Social Studies and Professor of Economics at the University of Essex.

Many and varied academic honours came his way. He was elected a Fellow of the Econometric Society in 1978 and was the recipient of Fullbright, Guggenheim and Erskine Fellowships. He co-authored four textbooks, edited a much cited and useful Penguin collection on *Monetary Theory*, produced two jointly written monographs on Liberia and Puerto Rico and published two volumes of collected essays.

In 1996, a handsome Conference to celebrate his 70th birthday was organised at the University of Trento and this resulted in a fine *Festschrift* with contributions from many of his close academic friends and admirers. I took the opportunity to give him, as his 70th birthday present, just before I gave my own talk at that memorable conference, what I thought was the perfect gift to a man who could not suffer fools, pretensions or pomposity: an original edition of Carlo Cipolla's pungent booklet, *The Four Basic Laws of Human Stupidity* (which was a personal gift from the author to me when Cipolla and I had been colleagues in Florence, more than two decades ago).

Bob Clower was an exceptionally generous man, particularly to Junior Colleagues and graduate students. I am a living example of this generosity, as his Falstaffian – 'brevity is the soul of wit' - contribution to my *Festschrift* (2010) would testify. I take this opportunity to mention this contribution only because I believe this was his last, written, professional paper.

Contrariwise, he could seem to be arrogant towards Senior Colleagues and anyone who exhibited the slightest trace of pretence or humbug. I have seen him 'carve eminent economists as dishes fit for the gods', at exclusive and specialized conferences, where sloppy analysis and incorrect historical knowledge were exposed

with razor sharp wit. His battles with Patinkin at such events were legendary and even entertaining to the cognoscenti.

The final academic move was to the University of South Carolina as the first Hugh C. Lane Professor of Economic Theory, in 1986. The move was motivated more by personal, family, considerations than anything else. By then the two daughters from his second marriage to Georgene were on the verge of becoming teenagers and Los Angeles, seething in all sorts of ways, did not seem to be an attractive place for family life. So this combative, extroverted, peripatetic academic chose the peace and tranquillity of Columbia, South Carolina, and literally at the supreme height of his academic career, withdrew to a 'quiet life'.

A debilitating stroke of a little over a decade ago seriously hampered Bob Clower physically, although it did not diminish in any way his mental alertness and agility till almost the very end. In his last years he was nursed with loving care by his wife, Georgene, with immaculate attention, at their beautiful Columbia home. It was only in the last few months that they moved to a small apartment, in the same attractive neighbourhood.

Clower had five children – four daughters and one son - from his first marriage to Frances Hepburn which was dissolved in 1975, and two daughters, Anastasia, born in 1978, and Kathryn, born in 1980, from his second marriage to Georgene Thousandfriend. He was always very proud, also, of his five grandchildren. All of them survive him.

§4. The *Art* of Monetary Macroeconomics – *after* Keynes

“I have always thought that the essential art of economics, as of any other science (or of literary fiction, for that matter) is to tell a good story in a persuasive way.”
Clower, 1984, p. 264

Clower's contribution to economic theory and methodology span vast, deep, wonderful and colourful canvasses. Monetary macroeconomics was his forte. But this was underpinned by a conviction, buttressed by solid theoretical work, and a recognition based on acute observation of real world phenomena, that economies

where money matters are those in which exchange and production processes can only be observed in disequilibrium states. Hence, he felt, it was imperative that an honest macroeconomic theorist should develop theories and construct thought experiments to elucidate such disequilibrium dynamics. With these convictions and understanding to guide him, he constructed those famous thought experiments of the *dual decision hypothesis* and the *dichotomised budget constraint* to suggest the kind of theories, models and concepts that were imperative for an understanding of monetary economies *in situ*. The concepts he introduced have become a part of the folklore of macroeconomics and the novice in his graduate studies meets them as ‘Clower Constraints’ or ‘Finance Constraints’, although it is only the form, not the economic content, that bears any relation to the creator’s concepts, as he has, himself, tirelessly repeated.

There was a particular novelty in the way he derived what came to be called the ‘Clower constraint’ which, in simplistic terms one can describe by saying that one cannot spend without having some money to do the spending with. The novelty was his explicitly stated awareness that going, by aggregating, from individuals to the macroeconomy, the only constraining microeconomic structural properties are Walras’s Law and continuity of the demand functions. These structural constraints, and their role in linking the individual and the collective, came to be formalized by the mathematical economists only some years later and now, generally, referred to as the *Sonnenschein-Debreu-Mantel theorem on excess demand functions*. One could, with perfect justice, add Clower’s name to the above trio, for no other economist has used it in the imaginative way he did, to draw out its deep implications in one particular and important domain.

The acuteness and sensitivity with which he has been an observer of the economic scene, and his scholarship of the classics of theory, convinced him, also, of the ubiquity of scale effects in economic processes. There are too many nonconvexities in production processes and too many indivisibilities in exchange processes, he claimed, over and over again. Monetary units, for example, do not come in infinitely divisible form. They come in units of whole dollars, euros or whatever and, *even in England*, in decimal subunits of the primary denomination.

He did not, like the uncritical curmudgeons of the profession, leave it at that; he went on, in several path-breaking papers, to try to construct with novel mathematical tools, rigorous models in which these non-convexities and indivisibilities could be explored theoretically. In one of these, jointly written with his younger collaborator and sometime colleague, Peter Howitt, both novel and difficult aspects of number theory and dynamical system theory were invoked, with the laconic caveat that ‘the proofs [of propositions about the transactions demand for money] necessarily involve the use of number theory – a branch of mathematics unfamiliar to most economists.’ This exercise was to illustrate another of his firmly held beliefs: that even simple issues in economics, in this case the familiar and almost banal case of the transactions demand for money balances, require careful analytical considerations and sophisticated mathematical tools to be faithful to real phenomena.

His co-author for the early book on an *Introduction to Mathematical Economics*, Bushaw – a noted expert on dynamical systems theory - was also his trusted lieutenant in his explorations into the difficult field of disequilibrium economic dynamics. It is, therefore, not surprising that his penetrating criticism of standard economic dynamics of varieties of Walrasian *tâtonnement* were technically well informed. This writer has benefited in numerous ways from Clower’s vast knowledge and almost perfect recall of mathematical classics. When I was working on the literature on topological fix point theorems and their applications in dynamical systems theory, it was to Clower I first turned for some guidance and he, unerringly, told me to begin with the papers by the elder Birkhoff on Poincaré’s ‘last conjecture’.

These considerations of ‘lumpiness’, he felt, rendered the welfare propositions of economic theory, particularly the two so-called *fundamental theorems of welfare economics*, which underpin many of the efficiency claims of theoretically derived policy proposals, inapplicable and even dangerous, if taught and retained in textbooks. He devoted the opportunities for reflection granted by Presidential addresses to the Western and Southern Economic Associations and to the History of Economics Society to propagate these themes in wise and measured ways. In these superbly crafted lectures he also called for a return to the *inductive traditions* of Adam Smith, William Whewell and, of course, Maynard Keynes.

§5. Monetary Macroeconomics *beyond* the Neowalrasian Code

“Hence, the present perplex in economic theory; for *if neo-Walrasian theory is bankrupt* – as, for practical purposes, it most surely is – then where do we go from here?”

Clower, 1984, p. 198; italics added.

Clower, in his Presidential Address to the *Southern Economic Society*, (Clower, 1994, pp. 805-6) noted with characteristically acerbic wit:

"Before I proceed, let me emphasize that by 'pure theory' I do not mean what most working economists ... mean when they use the word 'theory' without further qualification. Generally speaking, we mean by 'theory' the fact-oriented creative mixture of intuition, casual empirical knowledge, and seat-of-the pants logic that is found in virtually all 'applied economic analysis' and, indeed, in virtually everything called 'economics' before 1950. ... By pure theory I mean the axiomatically-based neowalrasian analysis of Arrow-Debreu .. . and closely related offshoots that serve as a standard of 'economic correctness' in all modern teaching not only in microeconomics but in macroeconomics, money and banking, finance, and econometrics."¹²

By 'economic correctness' Clower means what Howitt referred to as 'the neowalrasian code':

"[A]dherence to an increasingly complex code of formal ideas has become the overriding criterion of success, rather than the fruitful modelling of observed phenomena. The code of modern economics has become for the most part that of neowalrasian analysis, with its rules for modelling all behaviour as the outcome of rational choice. But accounting for some phenomenon in a discipline dominated by an elaborate code consists not of telling stories designed to convince others that this is why the phenomenon exists, or why it appears the way it does, but of telling stories, no matter how *ad hoc*, that incorporate some aspect of the phenomenon, no matter how trivial, *without violating the code*.

¹² To this must be added Clower's even more acid remarks on the pseudo mathematical formalisms of orthodox mathematical economics (in the draft version of what became Clower, 1994) :

“Hahn in his 1973 inaugural lecture [*On the Notion of Equilibrium in Economics*] seems to confuse 'finite' with 'relatively small'. Otherwise, why does he see an 'empirical confrontation' between Arrow-Debreu and fact only in an uncertainty model where 'markets exist at every date'? The product of two finite numbers is finite. Suppose in a certainty model the finite number I is 10^{17} . I doubt if the world contains that many spot markets. So what? The world probably contains even fewer ufs. If books are to be written about 'missing markets' on the basis of Hahn's 'empirical confrontation', I suggest a volume or two on 'missing' ufs.”

I suspect 'uff' refers to ufo's. To this impeccably acute observation I should add, by way of substantiation, a personal example. During a recent seminar in my department, one of my senior colleagues, who prides himself on being the 'resident theoretical econometrician' and an acknowledged and proud practitioner of neoclassical economic theory remarked something like the following: 'All these results are asymptotic results; therefore, 30000 observations, being closer to infinity than 10000, are to be preferred'! The incredulity with which I looked at him did not, I do not think, faze him one iota.

In many ways, modern economic theory hasbecome a purely logical discipline in which the objective is to follow a set of a priori rules with no connection to the external world. Economists building `rational models' to account for things not found in conventional theory think of themselves as seeking explanation in the usual sense, whereas in fact they are just addressing *purely semantic questions* that do not even arise once one ventures out of the *neowalrasian cloister*. Only by the rarest fluke could someone working under such delusions come up with a convincing scientific explanation of anything."
Howitt, 1996 pp. 75-6; italics added.

I have come to believe that the Clower's vision for an analytical basis for monetary macroeconomics was a formal framework which could encapsulate *Diophantine Dynamics*. This is an inference based on trying to extract a common theme from the *15 Clowerian Codes*, outlined above, in §2. The integral valued constraints with which Clower developed his outstanding and innovative model of the transactions demand for money (jointly with Howitt, in Clower-Howitt, 1978), the importance he came to attribute to non-convexities via 'lumpiness' in production processes, the irrelevance of the two fundamental theorems of welfare economics under such conditions and, a fortiori, in disequilibrium dynamic monetary economies, are all important elements that lead me suggest that the notion of *Diophantine Equations*¹³, within the framework of *Decision Problems* (in the strict sense in which this is interpreted in metamathematics, for which see Velupillai, 2011) is the best way for a path 'beyond neo-Walrasian economics'. Diophantine Equations, formalising economic relations – between more-or-less rational agents, in empirically meaningful Institutions – under integral valued constraints for values and quantities are typically only satisfiable, in the classic Herbert Simon sense, within the framework of *Decision Problems*.

In his concluding '*Reflections on the Keynesian Perplex*' (Clower, 1975, p.18):

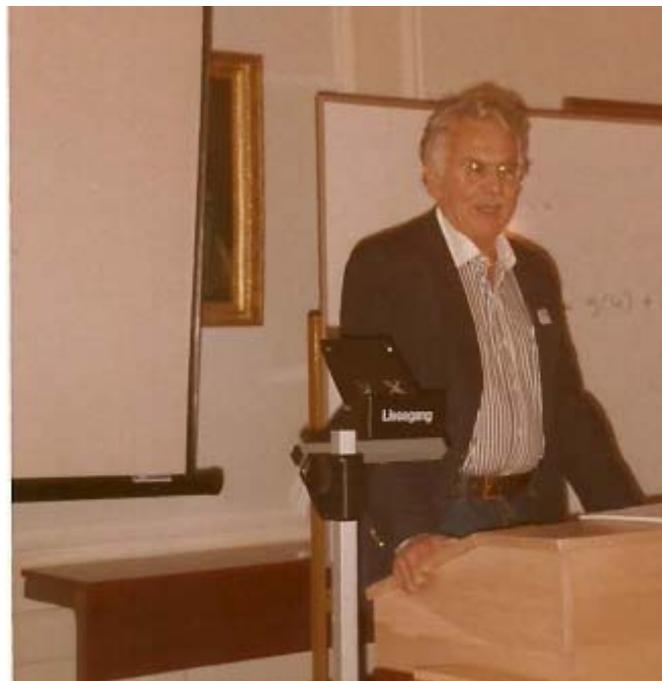
“[I]t is fallacious to argue that standard theories of individual or market behaviour can serve directly as a conceptual basis for models that describe the dynamics of disequilibrium motion; in particular, it would be wrong to regard established microtheory as a suitable foundation for macrotheory, for the central if not the sole object of macrotheory is to enhance our understanding of short-run disequilibrium adjustment processes.”

Clower's lifelong contributions, from the classics of the *annus mirabilis*, to his final reflections, were an attempt to contribute to the vision suggested in these obviously

¹³ Note: *not* inequalities.

Keynesian reflections – which also happen to place him squarely in that kind of behavioural economics fostered by Herbert Simon¹⁴. That which was missing in the latter’s pioneering contributions was money and monetary institutions; the former, although implicitly about behavioural foundations for monetary economics, did not develop a formal framework for it. Clower’s work bridged the gap – with a little help additional help from Alfred Marshall, Donald Bushaw, Charles Sanders Peirce, and Norwood Russell Hanson.

§6. Appendix: Number Theory, Recursion Theory, Integer Programming and Diophantine Economics



Bob Clower Lecturing at the First *Computable Economics* Summer School, Aalborg, 1990

In their remarkable model of the *Transactions Theory of the Demand for Money: A Reconsideration* (JPE, 1978)¹⁵, Clower and Howitt, imposed integer constraints on all economic variables, particularly monetary variables, and on all trading dates. As a result their proofs necessitated (footnote 3, p. 452; italics added):

“...[T]he use of number theory -- *a branch of mathematics unfamiliar to most economists.*”

¹⁴ See Clower, 1963, p.190.

¹⁵ The UCLA Discussion Paper version (Discussion Paper, Number 72, June 1976), titled: *Money, Credit and the Timing of Transactions*, is complete in the mathematical details and proofs, which are only sketched in the published, JPE version.

Gauss, often referred to as the Prince of Mathematicians is reputed to have considered Mathematics the Queen of the Sciences and Number Theory, in turn, the Queen of Mathematics. If two eminent economic theorists, both well versed in the mathematics of economics, proclaim that this 'branch of mathematics' is 'unfamiliar to most economists', then what can we hope?

We can try to become *computable economists*! As suggested in the last section of this paper, *Clower was a computable economist before his – and its - time.*

Rózsa Péter's opening lines in her classic text on **Recursive Functions** (1967), noted:

"The theory of recursive functions properly belongs to number theory; indeed, the theory of recursive functions is, so to speak, the function theory of number theory. ... The notion of recursive function marks off those functions whose values can be effectively calculated at every particular point; and just those functions are useful in the natural sciences. Though the variables of recursive functions do not run through all real numbers but only the natural numbers, probability theory as well as quantum theory operates with functions of this latter kind; and recently recursive functions have begun to be applied in analysis too."

The Clower-Howitt mathematical framework is, in its essence, is one of *integer programming*. Such problems are *Diophantine Decision Problems* (see, Velupillai, 2009).

Number theory, recursive function, integer programming and Diophantine Decision problems circumscribe the kind of mathematical economics lying at the core of Clower's economic theory. He was far ahead of his time and the implications of Clower-Howitt (1978) are yet to be worked out in their natural settings: as problems in *Diophantine Economics*, a field yet to be born.

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