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GOODWIN IN SIENA
ECONOMIST, SOCIAL PHILOSOPHER AND ARTIST

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ABSTRACT

This article describes the years of Goodwin's 'third life' at the University of Siena, stressing in particular his main contributions to economic theory, the evolution in his political and philosophical ideas and his love of art. First, after a brief description of his arrival in Siena and of the background he found there, we explore some of his contributions of the period and we argue that his achievements in the explanation of irregular growth cycles are remarkable and still open to further analysis and generalisation. Second, we remark that, as testified by an unpublished essay, in these years of dramatic changes such as the Fall of the Berlin Wall, his social philosophical beliefs underwent a significant evolution. Finally, we conclude with a tribute to Goodwin, the abstract painter.

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1. Introduction

In what follows we offer an overview of the period that Goodwin spent in Siena after his retirement from Cambridge University. The three aspects that have always characterised his life, which are epitomised in the title, are present also in his ‘third life’, as he used to label the Sienese period. First of all, we believe that his scientific accomplishments of the period are noteworthy in that he managed to provide a single unifying theory of irregular growth cycles, a goal he strived to reach throughout his life. The description of this task occupies most of the paper. At the same time, he acutely observed the dramatic changes that were occurring before his eyes in the political and social aspects of the world he had known from his youth and that formed such a large part of his personality. The fall of the socialist countries precipitated his reflections on social philosophy and to this aspect, as reflected in an unpublished essay, we devote a section of the text. Finally, he was able to pursue his artistic inspiration and he had his talent publicly recognised with an exhibition in Siena and the publication of a critical study to which we refer in the final section of the paper.

2. Background

The arrival of Goodwin in Siena was one of the unintended consequences of the reform of the Italian university system approved by Parliament in 1980. For the first time it was possible for foreign scholars to enter a national competition for a professorship. Until then foreigners had only been able to become professors by means of an *ad hoc* procedure. Goodwin had just retired from Cambridge University but, luckily enough, in Italian universities you could teach until 75. Some members of the Faculty of Economics and Banking (now, the Faculty of Economics ‘Richard M. Goodwin’), in particular Ugo Pagano (see Punzo in Di Matteo, 1990, p. 32), urged him to participate in the competition and actually helped him through the bureaucratic muddle. Goodwin, at the age of 67, applied and, not surprisingly, won one of the available chairs in *Economia Politica*.

The presence of Goodwin in Siena was a truly unique event and not only because of the circumstances just mentioned. He was already well known in Siena and had been invited to give several seminars in the early 1970s, attended also by several students, some of whom happened to be part of the faculty in Siena a decade later. He was very happy to be in Italy, a country that he had loved both for the art treasures and life-style he had been acquainted with since his first visit in the 1930s.

We do not know whether Goodwin really thought that Siena would be a place to enjoy, undisturbed, without too many commitments, a high quality of life, as he sometimes stated. Reality, however, turned out to be different. He was highly respected by a group of young colleagues (among which one of the present authors) who pressed him to debate with them major themes in economics, politics and social behaviour. Probably flattered, he actively participated in the discussions and offered a pleasant setting where we used to meet. He had rented a beautiful house in the countryside near Siena and introduced the tradition of the "Open House": every Thursday evening, we were invited to join him in front of the fireplace for endless discussions with some food, Chianti wine and music.[1] When the weather was nice, some of us were also invited to take part in a ride in his Fiat Dino in a search for new *osterie* nearby.

We were fascinated by listening to him and by his experience unfolded in crucial years, from the Great Depression to the post war period, and intertwined with so many giants, from Schumpeter to Wiener. We liked his being unconventional and open to new ideas, and his belief in a society of free men and women ruled not by the strictures of economic power but by the force of ideas.

Goodwin was very active also on the teaching front from the moment of his arrival in Siena. He was asked to teach – in Italian! – a course in *Economia Politica* for students of the final year (among which one of the present authors) and immediately produced a set of notes (1980A) that were to form the basis of the first part of the book jointly written with Lionello F. Punzo, a young member of the faculty, entitled *The Dynamics of a Capitalist Economy* (1987), so providing a grand synthesis of multi-sector structure, dynamics and history. He was also involved in teaching in the PhD program that was to start in those years, another child of the reform we mentioned above.

At about the same time, two other important events were organised. First, Punzo, with the enthusiastic help of a small number of graduate students (among which again one of the present authors), arranged an enlarged Italian translation (Goodwin, 1982B) of his *Essays in Economic Dynamics* (1982A) at the same time as the English edition appeared. Second, in the Fall of 1982, a long and passionate interview, due to the competent effort of Maura Palazzi (1982), covering all of his scientific and personal life up to that moment, appeared with the bibliography of his works (for a complete bibliography, produced by one of the present authors with the help of the other, see Sordi, 1999).

This is the background against which one has to set the remarkable scientific achievements of his 'third life'.

3. A single theory of growth and irregular fluctuations

The topic on which Goodwin concentrated a large part of his effort while in Siena was that of the evolution of capitalism, with the aim of finally arriving at a 'truly unified, single theory of

[1] For reminiscences about the same tradition during the Harvard period, see the piece by Leontief in Di Matteo (1990, p. 13).

growth with fluctuations' (1990C, p. 97). As Samuelson aptly noted (Di Matteo, 1990, p. 17), he was like Euler: 'all his life returned to basic topics, spiralling upward at a deeper level.'

He had arrived in Siena with his personal approach to the study of the dynamics of the capitalist system based on the work of three great economists: Marx, Keynes and Schumpeter (hence MKS). He had also strengthened his long-standing idea that such a theory has to take into account the irregular nature of cycles: this was so important that in the just-quoted interview (Palazzi, 1982, p. 38), he expressed the opinion that no economic cycle really exists, which was rather surprising for someone who had spent all his life on this problem.

However, the path that led him actually to accomplish the task was nonlinear (could it be otherwise?) and wandering. In his approach to the final result (reached in a series of papers at the end of the 1980s/beginning of the 1990s) we can identify four successive steps.

3.1 The MKS system[2]

The first step consists in a thorough recognition that his vision is a blending of his own construction of the theories elaborated by MKS. This was prompted by the well-known circumstance that the year 1983 linked the three economists together. In that year, several conferences were organised to mark such a coincidence and Goodwin took one of these occasions (the conference held in Groningen in September 1983) to clarify how a theory could be constructed building upon insights inspired by MKS (1986C; for a comprehensive summary of his view on the MKS system, see also Part I in Goodwin and Punzo, 1987). The attempt is remarkable as the political visions of MKS were rather different

From Marx he did not take the labour theory of value, but the presence and role of the Industrial Reserve Army and the accumulation of profits as the essence of capitalism (see Goodwin, 1984A, p. 5). But, contrary to Marx's prediction, capitalism is still alive and it was Schumpeter who clearly saw that innovation in the production side is the driving force of the system and the ultimate determinant of the continued existence of a positive profit rate. However, even Schumpeter's theory was partial. He consistently refused to give away the idea that the economy is in a full employment situation and to consider instead the central role of aggregate demand in determining output. This was on the contrary emphasised by Keynes who did recognise the widespread occurrence of unemployment and completed his theory with the multiplier analysis elaborated by Kahn. The dynamical trajectory of national output follows the aggregate demand constraint, but the engine of the trajectory itself is profit and accumulation made possible by innovation: profits continually evaporate and recreate due to the working of competition.

Capitalism is incapable of growing steadily: as cost-reducing innovations are adopted following the search for profit by firms, a process of growth starts in a particular sector and

[2] It will not escape the attention of the informed reader that 'MKS system' is also the acronym for the measurement system used in physics.

then spreads to the whole economy via the multiplier/accelerator mechanism.[3] The self-reinforcing growth eventually becomes constrained by resources and in particular labour shortages. Although labour supply does respond to economic stimuli, it does so rather slowly so that it becomes an insuperable barrier (in spite of labour liberated by technical progress and natural growth). At this point growth decelerates and the whole cumulative motion works in reverse and becomes stable due to the presence of given expenditures, until a new innovation arrives large enough to kick the economy off the bottom. Innovation in cost-saving processes or new products, needless to say, is stimulated by the low level of profits typical of the depression. And although Schumpeter and Marx tell us that such an innovation will certainly arrive, it will be a new thing every time, giving to the process just sketched a historical specificity. To this broad picture one can add the fact that as the economy approaches full employment, real wages rise and finally squeeze profits which is another element accompanying the upper turning point. The specific nature of the innovation helps in deriving the nature of the fluctuation in growth rates: there will be long or short cycles depending on how big the innovation is.

So far so good but can we go beyond a description and build a simplified model that exhibits these features? What follows is devoted to a description of the next steps in his attempt to answer this question.

3.2 Irregular fluctuations in a deterministic model of the cycle

The second step is when, during his search for the final model, Goodwin got attracted by a new mathematical tool which was starting to become popular among economists, namely, chaos theory. The explanation of how this happened goes like this. In March 1985, Richard Day, one of the pioneers of the applications of chaos to economics (see, e.g., Day, 1982), arrived in Siena to work with the mathematician Giulio Pianigiani and agreed to give a seminar. To listen to Day was for Goodwin a bit like a 'religious conversion' (1993C, p. 306; see also 1991B, pp. 424-5). At that time, he was in a happy period of his life. By studying the multiplier-non-linear accelerator interaction, he had discovered a new (two-stroke) oscillator and, by using Volterra's prey-predator system, solved the 'growth cycle problem' on which he had spent most of his academic life. This happy state of mind, however, persisted only until his participation in Day's seminar. Although Goodwin was already fully aware of some of the results in chaos theory applications from his last years in Cambridge, he thought 'it was for pure mathematicians' (ib.). Listening to Day however, convinced him that 'some large parts of the irregularity evident in economic behaviour are to be explained by this fascinating new tool' (ib.) and, in the following years, he did 'little else but work on economic forms of chaos' (ib.) He had already begun to equip himself with the tools, concepts and techniques implied

[3] As stressed also by Velupillai (1998, pp. 8-9), here Goodwin should perhaps have inserted the name of Harrod as a fourth major economist.

by the on-going developments in complex dynamics and, as a result, the time delay between Day's seminar and Goodwin's first models with chaotic dynamics was very short. In retrospect, this appears hardly surprising, particularly given that Goodwin's first contribution to economic dynamics (1946) was on the irregularity of economic cycles: after forty years, he thought he had finally found a solution to the problem of accounting for the irregularity of economic time-series in a deterministic macrodynamic model.[4] Since then, and until the end of his life, he continued to produce models capable of generating complex dynamics.

To the best of our knowledge, the first model of this type elaborated by Goodwin is the one presented in the article 'The discrete charm of erraticism' (1989B, 139-56). Although first published in 1989, Goodwin had started to work on it much earlier as is testified by a typescript in our possession, dated December 1985, containing a first draft of the paper.[5]

To describe, briefly, its contents, we can say that in it the stress is on asymmetric oscillators, with the only non-linearity of full employment. Various versions of the same model are introduced. For all of them, the description of the resulting dynamics is not only verbal, but is also illustrated by simulations performed with *Phaser*, a software tool of which Goodwin must have been one of the first and most affectionate users.[6] Here comes a distinctive feature of him. He was already 73 years old when he started using *Phaser*. Personal computers were still in their infancy, not even equipped with hard disks, and the only way to perform simulations was by using two floppy disks, one for data, and the other for the software. Yet, with great patience and a thirst for learning, he engaged himself in discovering the mysteries of the program, to obtain, via a satisfactory combination of plausible parameter values and suitable initial conditions, the searched-for dynamics.

The crucial role in the dynamics of the model is played by two time-lags: a *rather short* one ($1/\alpha$) in the adjustment of output to demand and a *much serious, longer* one ($1/\beta$, $\beta < \alpha$) in the adjustment of capacity to output. Thus, given output Q, desired capacity K, real investment I, all other real autonomous demand A (constant) and total demand Q_d, the dynamic system of the model can be written as:

$$\Delta Q_t = \alpha (Q_{dt} - Q_t) \tag{1}$$

$$\Delta K_t = \beta (Q_t - K_t) \quad 0 < \beta < \alpha < 1 \tag{2}$$

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4 The continuity between his 1946 contribution and those of the Sienese period is confirmed by his choice of reproducing the former together with the latter in Goodwin (1989B).

5 A revised version of the typescript, which is also in our possession, is dated May 1987 and is identical to the final paper then published in 1989. Other early contributions of the same type are in Goodwin and Punzo (1987, pp. 106-25) and Goodwin (1988B), the second containing the lecture Goodwin gave in Modena in February 1986, at a Conference organised to celebrate his 73th birthday.

6 *Phaser* is a software for the simulation of dynamic system which was pioneered and authored in the mid-1980s by Hüseyin Koçak at Brown University. The first edition of the book containing the floppy-disk of the software was published in 1986. This explains why the figures in the first 1985 draft of Goodwin's paper are hand-drawn, whereas some of them are computer-generated by *Phaser* in the final 1987 draft.

where Δ is the first difference operator and $0 < a, b < 1$, such that $a + b < 1$.[7]

Writing $\kappa > 0$ for the capital-output ratio, real investment is given by:

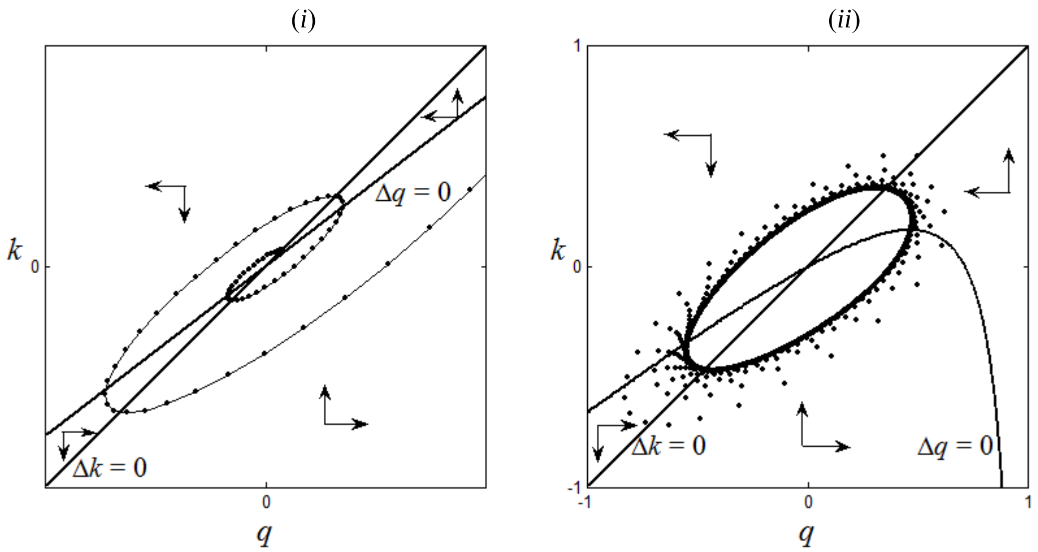
$$I_t = \kappa \Delta K_t = \kappa\beta(Q_t - K_t) \tag{3}$$

Inserting (3) into (1), and using small letters to indicate deviations from the equilibrium values $Q^e = K^e = A/(1 - a - b)$, (1) and (2) become:

$$q_{t+1} = \{\alpha[\kappa\beta - (1-a)] + 1\}q_t - \alpha(\kappa\beta - b)k_t \tag{4}$$
$$k_{t+1} = \beta q_t + (1-\beta)k_t \tag{5}$$

The coefficient matrix A of system (4)-(5) is such that $Tr(A) = \alpha(\kappa\beta + a - 1) + 2 - \beta$ and $Det(A) = \alpha(\kappa\beta + a - 1) + 1 - \beta + \alpha\beta(1 - a - b)$. Assuming that $\kappa\beta > 1 - \alpha$, from (4) and (5) it follows that, in the phase plane (q, k), the $\Delta q_t = 0$- and $\Delta k_t = 0$-isoclines are both straight lines with slopes equal to $0 < [\kappa\beta - (1 - a)]/(\kappa\beta - b) < 1$ and 1, respectively. For plausible parameter values (e.g., $a = 0.75$, $b = 0.1$, $\beta = 0.3$, $\alpha = 0.7$, $\kappa = 2.5$, such that $Tr(A) = 2.0500 > 0$, $Det(A) = 1.0815 > 0$, $(Tr(A))^2 - 4Det(A)) = -0.1235 < 0$), the model exhibits cyclical fluctuations of increasing amplitude around the unique equilibrium at the origin of the axes (see Fig. 1(i)).

Fig. 1. (i) Fluctuations of increasing amplitude in the linear model and (ii) quasi-periodic fluctuations in the model with the hyperbolic barrier at full employment



[7] The notation may appear a bit confusing and for this reason it is useful to stress that K indicates the (desired) maximum amount of output that it is possible to produce in a given period of time. It is in other words a *flow* variable which, according to (2), is increased or decreased so as to reduce the discrepancy between its value and realised output.

We must take account of the fact that there is a crucial constraint for any unstable dynamics of this type, given by the impassable barrier of full employment output (q^*); in other words, the oscillatory mechanism we have just described is *asymmetric*. For this reason, Goodwin (1989B, pp. 148-51) rewrites equation (4) in such a way that it embodies a *hyperbolic* barrier at q^*. This is done by simply adding a term on the right-hand side of the equation, with the difference $\left(q^* - q_t\right)$ in the denominator such that the dynamic system becomes non-linear:

$$q_{t+1} = [\alpha(a + \kappa\beta - 1) + 1]q_t - \alpha(\kappa\beta - b)k_t - \gamma\left(\frac{1}{1 - q_t} - 1\right) \tag{6}$$

$$k_{t+1} = \beta q_t + (1 - \beta)k_t \tag{7}$$

where $0 < \gamma < 1$and where q^* is normalised to one.

The origin of the axes is still the only economically meaningful solution of the system. We may examine the dynamics generated by the latter by studying, first of all, its Jacobian matrix J. The elements of J evaluated at (0,0) are the same as those of the coefficient matrix A of the linear system (4)-(5), apart from the element J_{11} that now becomes:

$$J_{11}| = \alpha(a + \kappa\beta - 1) + 1 - \gamma$$

so that we now have $Tr(J) = \alpha(a + \kappa\beta - 1) + 2 - \beta - \gamma$ and $Det(J) = \alpha(a + \kappa\beta - 1) + \beta\alpha(1 - a - b) + (1 - \gamma)(1 - \beta)$. Thus, the (local) stability conditions become:

$$1 - Tr(J)| + Det(J)| = \beta\alpha(1 - a - b) + \beta\gamma > 0 \tag{8}$$

$$1 + Tr(J)| + Det(J)| = 4 + 2\alpha(a + \kappa\beta - 1) + \beta\alpha(1 - a - b) - 2\beta - 2\gamma + \beta\gamma > 0 \tag{9}$$

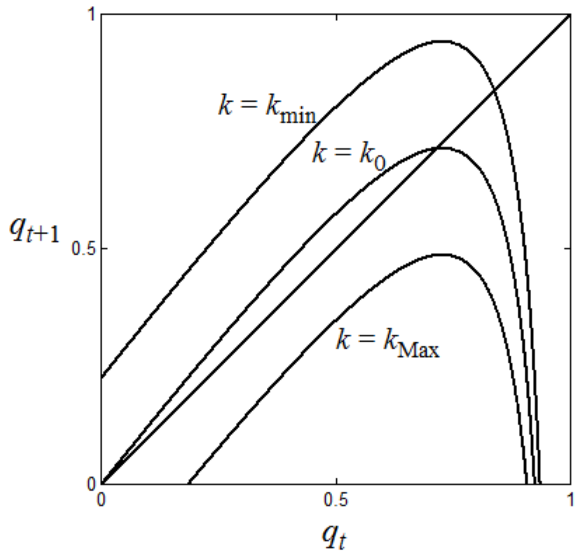
$$1 - Det(J)| = -\alpha(a + \kappa\beta - 1) - \beta\alpha(1 - a - b) + \beta + (1 - \beta)\gamma > 0 \tag{10}$$

We immediately see that condition (8) is always satisfied. Moreover, with the same parameter values we have used in the previous example, and $\gamma = 0.1$, it is easy to check that (9) is also satisfied whereas (10) is not, and that $(Tr(J))^2 - 4Det(J) < 0$. Thus, the equilibrium point is locally unstable. Numerical simulation shows that it is now surrounded by a stable limit cycle, representing persistent (*quasi*-)periodic fluctuations of the two variables (see Figure 1(*ii*)).

Finally, Goodwin, as in several other early applications of chaos theory to economic modelling, reduces the dynamic system of the model to an equation of the logistic type. In his case, this can be done by simply remembering that output adjusts much faster than capacity to desired levels. Thus, as a first approximation, it is possible 'to look at the rapid variation of q, with fixed k (as Keynes did in the *General Theory*)' (1989B, p. 151).

For a given value of $k_t = k$, system (6)-(7) reduces to:

Fig. 2. Graph of the function $\phi(q_t;k)$,for different values of k
($k_{\min} = -0.5$, $k_0 = 0$ and $k_{\text{Max}} = 0.5$)



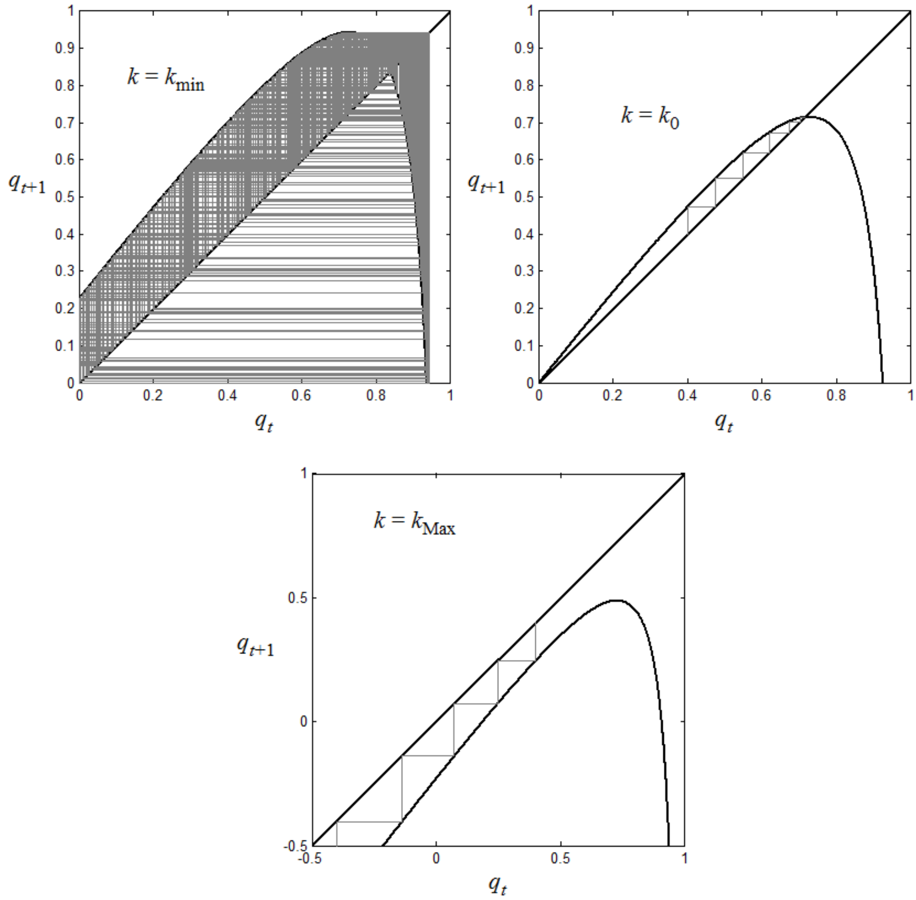
$$q_{t+1} = \phi(q_t;k) = \left[\alpha(a + \kappa\beta - 1) + 1\right]q_t - \gamma\left(\frac{1}{1-q_t} - 1\right) - \alpha(\kappa\beta - b)k \tag{11}$$

where $\phi(q_t;k)$ is a unimodal function as shown in Figure 2 for different values of the *parameter k*. In the case in which k varies between a given minimum value and a maximum one, the curve is lowest for k_{Max} and highest for k_{min}. The curve sinks with increasing values of k and, during this process, striking bifurcations occur. Some examples are shown in Figure 3, where with all the other parameters unchanged, we see that the dynamics of the model can be chaotic, convergent to the equilibrium point or monotonically unstable downward (cfr. Sordi and Vercelli, 2006, pp. 426-8).

3.3. Long waves

To understand the third step in Goodwin's attempt to build a theory of oscillating growth we must keep in mind that the problem of providing a unifying theory of it appeared to him as equivalent to explaining long waves (or *Kondratiev cycles*, in Schumpeter's terminology). It must be remembered that in the early 1980s there was a renewed interest in long waves following the generalised crisis that began with the first oil shock and that was intensified by the second (see van Duijn, 1983). As noted by Velupillai (1998, p. 26), he had dismissed the idea of long waves during the Cambridge period, but it was time for him to go back to the topic and to Schumpeter. In the first attempts to deal with Kondratiev cycles, Goodwin built

Fig. 3. Iteration of the map for different values of k
($k_{\min} = -0.5$, $k_0 = 0$ and $k_{\text{Max}} = 0.5$)



upon his previous work seeking to combine in a different way old analytical tools and new insights.

Two occasions (a conference in Weimar in June 1985 and a workshop in Siena in December 1986) induced him to advance a preliminary theory of long waves (see Goodwin, 1986B; 1987C; 1989B, pp. 70-89 and 1986E; 1989A; 1989B, pp. 45-61). In a sense, this can be seen as a return to his origins in the field: he recognises indeed that Schumpeter's insights are the most valuable ones. After all, if growth and cycles are to be studied together, as the latter is the form the former takes in a capitalist economy, it is natural to see the whole process as a series of Kondratiev cycles.

In the search for a theory, he divided his task into two parts in the attempt to understand, first, how a single long wave arises and, second, how the completion of each wave leads to a situation which produces another wave. His starting point is the cyclical process given by the interaction between the distribution of income and the labour market dynamics via the Phillips curve, jointly with a rule governing the pattern of output − what he defines as a

Dynamical Say's Law – and a rule governing the pattern of labour productivity and labour force. Assuming that all wages and no profits are consumed, the former can be written as:

$$q = \left(a_q + a_l w\right) q + \left(a_q + a_l w\right) q' \tag{12}$$

where a prime after a variable indicates first derivative with respect to time, a_q and a_l are the input of goods and of labour per unit of output, respectively, and w the real wage. Moreover, indicating with u the share of labour, we have:

$$u = \frac{lw}{q} = \frac{a_l q w}{q} = a_l w$$

where l is employment. Thus, (12) becomes:

$$\frac{q'}{q} = \frac{1}{a_q + u} - 1 \tag{13}$$

The price of labour depends on scarcity, which Goodwin measures by the employment rate, v, defined as the ratio between l, and the available labour force, N; thus:

$$\frac{v'}{v} = \frac{l'}{l} - \frac{N'}{N} = \frac{q'}{q} - \left(-\frac{a_l'}{a_l} + \frac{N'}{N} \right) \tag{14}$$

Finally, writing the Phillips curve as:

$$\frac{w'}{w} = \phi(v) = \mu\left(\frac{1}{1-v} - \theta\right), \quad \phi'(v) > 0,\ \theta > 0 \tag{15}$$

we have:

$$\frac{u'}{u} = \frac{a_l'}{a_l} + \frac{w'}{w} = \mu\left(\frac{1}{1-v} - \theta\right) + \frac{a_l'}{a_l} \tag{16}$$

With exogenous rates of growth, such that a_l decreases exponentially in time and N increases, the dynamic system in v and u formed by equations (14) and (16), jointly with equation (13), produces a wave-like movement of v and u, accompanied by a growth cycle in q.

The growth of productivity is then endogenised by Goodwin by assuming that it occurs 'only when there is an increase in investment in innovative technology' (ib., p. 53). More precisely, he takes the rate of change of the labour input as proportional to a logistic growth in innovative capacity where, to take account of Schumpeter's vision, capacity generated by the cluster of innovations (k) is assumed to follow a logistic path in time of the type:

$$k' = \varepsilon k\left(1-\frac{k}{\sigma}\right), \quad \varepsilon,\sigma > 0 \tag{17}$$

This represents the full absorption of innovation into the economy such that innovative investment approaches zero as capacity approaches its saturation level σ. Thus, we can write:

$$\frac{a_l'}{a_l} = -\varphi\frac{k'}{k}, \quad \varphi > 0 \tag{18}$$

But this cannot be the end of the story as, following Keynes, investment in innovative capacity acts positively on output growth, such that (13) becomes:

$$\frac{q'}{q} = \frac{1}{a_q+u} - 1 + \kappa\varphi\frac{k'}{k}, \quad \kappa > 0$$

Putting all together, and taking for simplicity a constant labour force, the dynamic system of the model is now formed by (17) together with the following two equations:

$$\frac{v'}{v} = \frac{1}{a_q+u} - 1 - (1-\kappa)\varphi\varepsilon\left(1-\frac{k}{\sigma}\right) \tag{19}$$

$$\frac{u'}{u} = \mu\left(\frac{1}{1-v} - \theta\right) - \varphi\varepsilon\left(1-\frac{k}{\sigma}\right) \tag{20}$$

The only positive equilibrium point is $(v^*, u^*, k^*) = (1 - 1/\theta, 1 - a_q, \sigma)$. To study its local stability, we consider the elements of the Jacobian matrix which, evaluated at (v^*, u^*, k^*), are given by:

$$J_{11} = 0,\ J_{12} = -\frac{v_1}{\left(a_q+u_1\right)^2} < 0,\ J_{13} = \frac{\varphi\varepsilon v_1}{\sigma} > 0$$

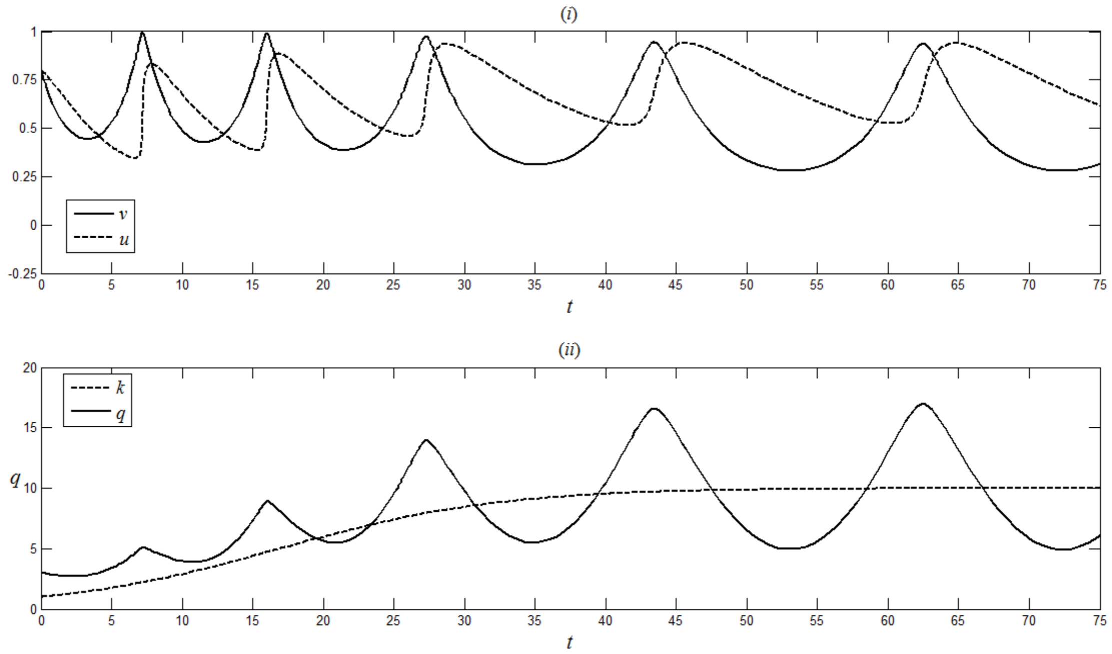
$$J_{21} = \frac{\mu v_1}{\left(1-v_1\right)^2} > 0,\ J_{22} = 0,\ J_{23} = \frac{\varphi\varepsilon u_1}{\sigma} > 0$$

$$J_{31} = J_{32} = 0,\ J_{33} = -\varepsilon$$

Thus, one of the eigenvalues is real and negative ($\lambda_1 = -\varepsilon$), whereas the other two are purely imaginary conjugate numbers, obtained as solution of $\lambda^2 - J_{12}J_{21} = 0$. Numerical simulation (see Figure 4) shows that the positive equilibrium point is a centre. From Figure 4(*ii*) it is moreover evident that, with a cluster of innovations such that k approaches its saturation level σ in about fifty years, the output grows through cycles for about fifty years and then starts to fluctuate around a stationary state until the next cluster of innovations.

What happens is that, as the innovation begins to have effect, it acts positively on output and negatively on cost of labour thanks to the increase in productivity. The rise in output is

Fig. 4. The dynamics of u, v, q and k in the model with endogenous growth in labour productivity



marked by several short oscillations that become more pronounced towards the end of the long wave. The slowing down of innovative investment due to the flattening of the logistic curve brings in the cessation of growth but the level of output per capita and real wages are higher than at the beginning. On the other hand, the employment ratio shows a moderate rise in the first part of the long wave and a significant fall as innovative investment stabilises to the saturation level.

Now comes the second part, namely how there may be a succession of long waves. Here is the innovative aspect of Goodwin's construction. The completion of a logistic investment process produces depressed economic conditions, and may leave a growing gap of unexploited possibilities. This will give rise to a renewed pulse of innovations in such a way that, 'in a sense, there is really no oscillation but rather a series of cumulative, large scale pulses' (1989B, p. 59). The widening of the gap between existing and potential technology will, sooner or later, be 'sufficient to initiate a new logistic pulse. [...] Here, unlike in the usual logistic, there is not a given, unchanged saturation level for innovations. [...] In this way one can combine two processes, neither of which is oscillatory, but together, if connected by a pair of threshold effects, will produce an alternating dynamic of growth and stagnation'(ib., pp. 59-61).

This way of approaching long waves gives a role to the historical aspects avoiding a mechanical construction which implies a given, unchanged mechanism of oscillation throughout a very long span of time (cfr. Velupillai, 1990, p. 15). What Goodwin actually produced was not a cycle but a sequence of oscillations. However, he was not fully satisfied as he could not replicate the irregular nature of the process. Although his work on long waves is of a period in which he had already discovered chaos – and although his analysis of the matter starts with the derivation of a simple discrete-time formulation capable of generating chaotic dynamics (1989B, 48-51) – the continuous-time versions of the model are unable to generate irregular long waves. The impression one gets is that Goodwin was still in search of the 'proper mathematical tool', which he will soon finally find in the Rössler oscillator.

3.4. Irregular fluctuations and the Rössler oscillator

The fourth and final step is when Goodwin realised that chaos theory, not in the form of the standard discrete-time applications to economics based on unimodal maps, but relying on higher order dynamic system in continuous-time, could help him.

He had always been eager to use his knowledge of physics to build economic models that capture the evolving nature of capitalism. This attitude derived not only from his curiosity towards hard sciences, but more profoundly from the belief that economic problems are akin to those encountered in physics and biology and require applied mathematics rather than elegant mathematical theorems. So he repeatedly searched for tools that could help him in this effort.

This search lead him, around the end of 1987, to discover the Rössler oscillator which he first mentioned in the notes he prepared for presentation at the First International School in Economic Research organised in Siena in November 1987 (Goodwin, 1987A).[8] These notes do not contain any model, but simply a discussion about the new tool.

The first attempt to build a dynamic model giving rise to a Rössler oscillator is contained in Part IV of Goodwin (1988A) (see also 1990C, pp. 48-55), where the starting point is again the model we have analysed in the previous section, with a logistic growth in innovative capacity. The dynamic system, without any explanation, is however now written as:

$$\begin{aligned} v' &= -u - k \\ u' &= v + au \\ k' &= b + k\left(v - c\right) \end{aligned} \tag{21}$$

which is a system exactly in the form proposed by Rössler. Goodwin sees it as 'very general and primarily useful for indicating the range of behaviour type, rather than specific economic system' (1989B, p. 133) and this is in fact what he does in the rest of his analysis. Thus, it appears that, although he was attracted by the Rössler oscillator as the simplest example of a

[8] These notes were then published, with some additions, as Part I of the lectures he gave at the European University Institute (1988A) at the beginning of 1988 and, with some other additions and modifications, in Goodwin (1989B, pp. 115-138) and (1990C, pp. 1-21).

chaotic attractor in continuous-time, he does not yet know how to obtain it in a dynamic model of the economy (see also 1993B).[9] But then he started to check whether a more satisfactory economic model could exhibit a chaotic motion of the same type as the original Rössler model and finally succeeded! His path toward the final model can be briefly described in the following way (see, e.g., 1989F; 1990A; 1990C, Ch. 8; 1991A; 1991B).

Underlying his analysis there is a pair of first order differential equations, with parameters such that their solution gives unstable oscillations. If a third non-linear differential equation, which defines a dynamic control parameter, is added to them, this determines a bounded region in state space, within which sooner or later all trajectories of the original two variables, are constrained.

Starting from the dynamic system in v and u of Goodwin's (1967) growth cycle model,

$$\begin{aligned} v' &= (A - Bu)v \quad A, B > 0 \\ u' &= (-C + Dv)u \quad C, D > 0 \end{aligned} \tag{22}$$

where A, B, C and D are combinations of the parameters of the model, and linearising it around the positive equilibrium points $(v^*, u^*) = (C/D, A/B)$, we obtain

$$\begin{aligned} v' &= -du \\ u' &= hv \end{aligned} \tag{23}$$

where v and u are now taken to indicate deviations from equilibrium and where $d = Bv^*$ and $h = Du^*$. A term, representing a kind of accelerator, i.e., the fact that 'high and expanding demand leads to further increase in demand and output' (1989F, p. 7), is added on the right-hand side of the first equation so as to obtain:

$$\begin{aligned} v' &= fv - du \\ u' &= hv \end{aligned} \tag{24}$$

When $f^2 < 4dh$, (24) generates unstable fluctuations of the variables which, however, are constrained by full employment. The novelty in these contributions is that, rather than defining an upper bound to the dynamics of the variables, Goodwin decides to take account of the impassable barrier of full employment by positing 'a control parameter which provides a growing downward pressure beyond a given high (positive) value and a growing upward one for low (negative) values.' (ib.) The dynamics of the latter is specified as:

$$z' = b + gz(v - c) \tag{25}$$

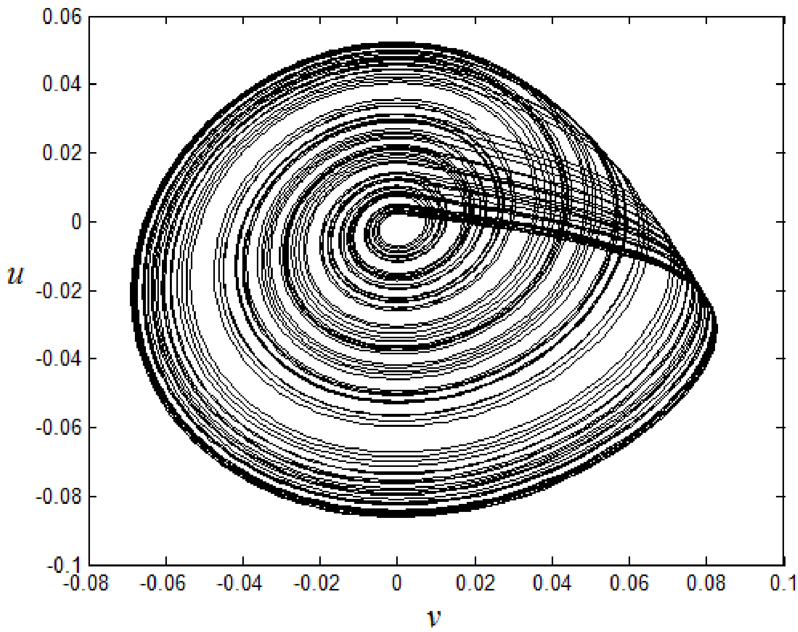
[9] His interest for the Rössler oscillator is best explained in the notes he prepared for presentation at the Conference of the International Economics Association (1989F), held in Copenhagen in June 1989, a shorter version of which was then published in the proceedings of the conference (1991B).

where z is the public budget surplus. What does (25) imply? We see that its right-hand side has a (small) exogenous component, b, and a second, more important, component, such that when the employment ratio is higher than a predetermined value c, the budget surplus increases and vice versa when v is less than c. The reason for this is that the changes in the budget surplus are the result of deliberate fiscal actions undertaken by the government: c in this case is the parameter within an idealised version of the derivative stabilisation policy of Phillips' (1954) classification, the aim of which is to reduce the amplitude of fluctuations. Adding (25) to (24), modified in such a way that there is a feedback between the dynamics of z and that of v, due to the fact that the budget surplus influences negatively the employment ratio by reducing aggregate demand, the system becomes a variant of the Rössler oscillator:

$$\begin{aligned} v' &= fv - du - ez \\ u' &= hv \\ z' &= b + gz(v - c) \end{aligned} \tag{26}$$

The dynamics is stabilised globally but, at the same time, it is now 'free to perform wildly erratic motion locally around the equilibrium' (ib., 8; see also Goodwin, 1990C, pp. 102-10; 1990A; 1991A). In Fig. 5, a projection of the resulting dynamics onto the (v, u) plane is shown, with initial condition (v_0, u_0, z_0) = (0.02, 0.03, 0.02) and d = 0.5, f = 0.15, e = 0.30, h = 0.50, b = 0.01, g = 85.0 and c = 0.05 as in Goodwin (1990C, p. 103).

Fig. 5. Chaotic dynamics of the Rössler type in the model with dynamic system (26)

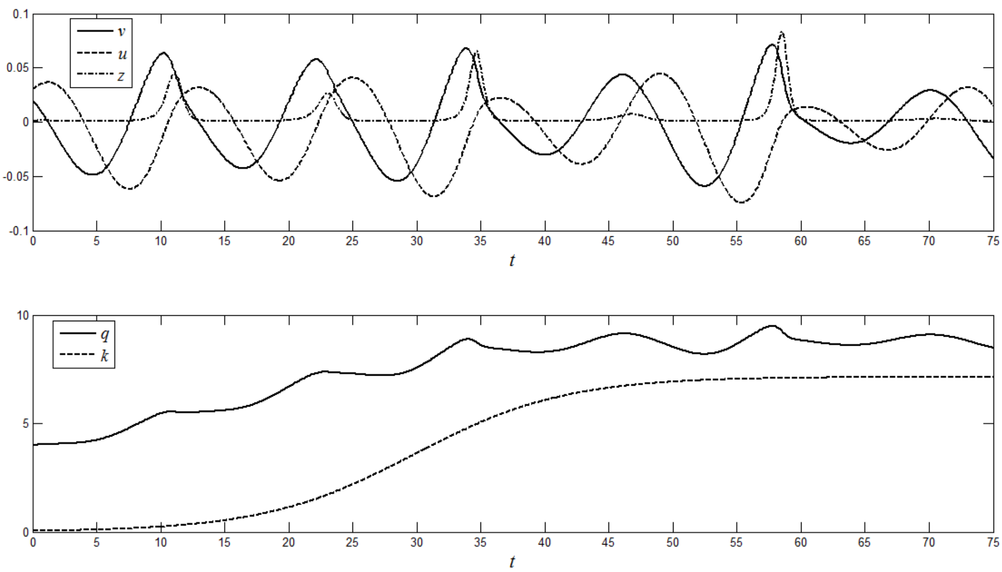


The link with the original Lotka-Volterra system, which in Goodwin (1967) was derived under seven simplifying assumptions, is now much weakened. As a result, he appears to feel free to modify the model further. Under the assumption of a constant labour force, when investment in innovative investment is as in (17) and productivity dynamics as in (18), the full system becomes:[10]

$$\begin{aligned}
v' &= fv - du - ez \\
u' &= hv \\
z' &= b + gz\left(v - c\right) \\
q' &= \left[\frac{fv - du - ez}{v + v^{+}} + \varphi\varepsilon\left(1 - \frac{k}{\sigma}\right)\right]q \\
k' &= \varepsilon k\left(1 - \frac{k}{\sigma}\right)
\end{aligned} \tag{27}$$

The projection of the resulting dynamics onto the (v, u) plane (this time with d = 0.5, f = 0.15, e = 0.80, h = 0.50, b = 0.0045, g = 85.0, c = 0.048, v^{+} = 0.90, φ = 0.16, ε = 0.17, and σ = 7.14 as in Goodwin, 1990C, p. 91), gives a picture very similar to that shown in Figure 5. For the present version of the model this in turn implies an irregular growth cycle in output as shown in Fig. 6.

Fig. 6. The dynamics of u, v, q and k in the final model



[10] Given that the dynamics in v and u is expressed in terms of deviations from equilibrium, a positive value v^{+} is added by Goodwin in the denominator of the first term on the right-hand side of the fourth equation in order to get a growth cycle rather than fluctuations around a stationary state.

The final model is then obtained by assuming that the increase in the innovative capacity k has two components, one as in (17), such that investment in k tend to zero as k tends to σ. The other, Goodwin argues, is due to the fact that investment not only influences demand and output but is in turn subject to the demand level, so that:

$$k' = (\varepsilon + nv)k\left(1 - \frac{k}{\sigma}\right), \quad \varepsilon, \sigma > 0 \tag{28}$$

The dynamics generated by system (27) with the last equation replaced by (28) and the same parameters as before with the addition of $n = 20$ is still of the same Rössler band type as in the previous two figures. Thus, Goodwin's final model appears very robust to variations in its structure and, as such open to further analysis and generalisation. Although here and there Goodwin is aware of the implications of chaotic theory for several aspects of economics we refrain here from even hinting at such implications. We would like only to mention that according to him econometrics should be reformulated in a profound way and that experimental economics could be a powerful tool to investigate actual behaviour of agents in relation to the standard hypotheses adopted by mainstream economists.

4. Art & social philosophy

But even in Italy university professors do retire[11] and on that occasion (March 1988) Siena organised a grand event appropriately to celebrate Goodwin with a conference attended by long time friends, mentors and students coming from all over the world, among which Samuelson, Leontief, Morishima, Leijonhufvud and Velupillai.[12]

As hinted in the Introduction, no description of the Sienese period would be complete without remembering that '(t)hroughout his life he has been simultaneously an artist, a political spirit and economist' (Di Matteo, 2000, p. 240).

4.1 The social philosopher

One has to register a significant evolution in the same period in Goodwin's political and philosophical ideas. And this, as we will argue, also has an interesting feedback on his conception of economics.

[11] Shortly after his retirement in March 1988, Goodwin donated to the Library of the Faculty of Economics a large part of his scientific and personal papers. The material has been organized in the Goodwin Archive and its catalogue put on the web (http://ase.signum.sns.it/ase), accessible to scholars worldwide. This was possible thanks to the interest in economists' archives by the *Società Italiana degli Economisti* and the funding by the Italian Ministry of Education and the Bank of Italy. For more information on the Goodwin Archive, the reader is referred to Di Matteo, Filippi and Sordi (2006).

[12] A booklet with the papers there presented was subsequently printed (Di Matteo, 1990). A piece by Stone who could not attend in person is also included along with a few touching words by Solow.

In the Goodwin Archive, there is the draft of a presentation, entitled *Sketch of a possible social philosophy*, prepared by Goodwin probably for a public talk that he gave in Siena in Spring 1991 (see Di Matteo, 2011). According to Goodwin, a new social philosophy is needed, one that is adequate in the present circumstances for the task of understanding the economic and social processes that are evolving so fast. A social philosophy can be defined as a general view and understanding of human and social life and of its evolution; it also includes a guide for action. An important part of this is the social code, namely that set of values and rules and norms that govern our behaviour. The new social philosophy that Goodwin appears to advance can be summarised as follows.

We are aware that change and innovation are the source of all the 'goodies'. This however should not mask the fundamental importance of continuity in history achieved *inter alia* via the transmission of the social code. In fact the latter is not immutable but subject to slow change and adaptation. The 'Romantic Revolution',[13] with its extreme emphasis on change, has led us to overvalue the new and the young at the expenses of a more balanced attitude. From this he drew two important conclusions: the end of Utopianism and the rejection of the revolution. By the first he meant that it is vain to search for an ever valid alternative to the existing arrangements of society. Societies evolve and therefore it is incorrect to believe that 'if there is a Right Solution then it is right for all time [...]. But in fact all such Right Ideas are historically conditioned and delimited in their applicability to a particular time and place.' His views are also anti-revolutionary as the continuity in the society is a vital ingredient of the social fabric. In his opinion, societies do evolve in the long run, sometimes more speedily, sometimes more slowly, but this is a more effective way of change than the abrupt alterations brought by revolutions that are doomed to failure. A positive explanation for the outbreak of revolutions is precisely that the required change of the social code has not been introduced at the right time. To summarize in a few words both implications: 'Society cannot be reconstructed *ab novo*, the possibilities are too great and we have no standard by which judge them. Rationality is no help [...]. We inherit an irrational, historically accidental situation and all we can do is modify it, not reconstruct it.'

Here we have not only the awareness, based on his evolutionary vision, of the historical failure of revolutions: there is a definitive theoretical criticism that such a Utopia can be a valid goal. The repeated criticism of Platonism contained in the essay is certainly not directed to the ideal of the dignity of the human soul but precisely to the idea that this Utopia should be the basis of a revolutionary movement.

There is a second set of implications from Goodwin's general view. The individualistic approach to economics, and other social sciences as well, is false. This derives from the recognition that the individual is unfree, his behaviour being controlled by the social code. This is a suggestion especially valid for economics which is the most important social

[13] By this expression he probably meant Marx, among others.

science. And it is an extremely innovative suggestion, a road that is now widely followed with the help of computer-based simulations. He placed himself at the frontier of economics in advocating a reformulation of economics based on his view of interacting agents. He certainly was not aware of what was about to start in those years but this is even more significant of a consonance of ideas and perspectives. Indeed, in the last twenty years, among mainstream economists, there has been an increasing interest in an evolutionary approach based on simulations of agents endowed with different norms of behaviour. According to Goodwin we have to refute a static vision of man and engage in a dynamic system analysis. Society must be looked at as a collection of thinking machines that interact with one another. Machines have a wide variety of behaviour depending on how they are programmed. Humans, although they have a genetic code, can have a very large variety of behaviour. The latter depends on the social code that '[...] builds into [them] the vast variety of individual behaviour which we rightly prize.' The social code in turn is shaped by education in the widest sense (family, school, social groups, etc.): this code, contrary to the genetic one, can and does vary, albeit slowly. As a consequence of the previous argument economic theory based on consumer sovereignty (i.e. given tastes, the only problem is how to satisfy them) is doubtful as it assumes that tastes are pre-social.

This theme is developed in Goodwin's short paper delivered in Rome in Autumn 1988 at the annual meeting of the *Società Italiana degli Economisti* and subsequently published as Goodwin (1989G). The thesis there expressed echoed Galbraith's famous dependence effect expressed in his *Affluent Society* (1958) but disregarded the severe criticism by Hayek and Wallich (for the terms of the debate, see Phelps, 1962).

Goodwin was smart enough to be aware that the evolutionary view to which he subscribes is liable to criticism as to historicism everything is flux, except that historicism itself is true: 'Hence it is a Russell Paradox: the historian who tells you that all historians are liars.'

In spite of the difficulties of his evolutionary view Goodwin grasps the nettle and embarks on a series of remarks with explicit reference to today's situation. Although the essay was probably written more than twenty years ago, we think that his observations maintain their interest.

What are presently the main features of the world economy in the era of the (now) so called globalisation? Due to the increasing division of labour there is a widespread interdependence among economies fostered and fed by a prodigious growth of international trade: the landscape is dominated by large multinationals run by a very restricted power elite. This feature of a unified world market populated by world actors is to be regarded as a permanent, not transitory, phenomenon, the reason being an increasing demand for these worldly goods coming from the so called Third World. The population there wants to reach the same standard of living that is enjoyed by people in the western rich states. Goodwin thinks that the distribution of goods (and welfare in general) is really unequal across the nations of the world and that it will prove impossible to avoid a redistribution, already in the medium run.

All this forces us to change our social philosophy radically. According to Goodwin the 'inherited political philosophy [...] of democratic control [...] will in the future be subject to attrition, if indeed it survives at all.' The real power of decision, both in private and public organizations, because of the complexity of the structure, will be in the hands of small, unaccountable elites. The difficulty in controlling them will not be lessened: indeed, the development of technology brought about by the expanding economy will require more expertise and knowledge than before and this will not be available to the mass of population.[14] The latter could well be consulted but this is a very different matter from possessing control over the crucial decisions.

On the other hand, the traditional answer by socialists (like myself, Goodwin acknowledges) to this situation, namely planning, is no longer a valid option, as the State too is run by small elites and has become so big as to be uncontrollable. One can probably hear the echo of the failure of the communist experience, when Goodwin writes: 'There is a real potential gain in having the broad lines of development of a society under the control of able individuals as opposed to the mindlessness of our free market scramble system; but there is a price to pay and a frightening danger involved.'

His pessimism, although somehow in contrast to his evolutionary view, comes out clearly in the last part of his essay. Goodwin after so many decades of active economic policy shows less confidence in the power of ideas as opposed to that of vested interests in getting to a satisfactory situation (cfr. Goodwin, 1990C, p. 102).

4.2 The artist

Siena paid an affectionate and warm tribute not only to Goodwin the innovative economist but also to Goodwin the abstract painter. In Cambridge in the 1960s he had a very creative period and he hoped to have more time to paint when in Siena. As has already been observed Goodwin's lectures were a piece of art crowded with beautiful and complex diagrams in which trajectories of the strangest forms appear in different colours.[15] Let Goodwin himself speak of his art: 'A picture must show somehow a combination of both form and colour. Colour makes you stop in front of it. It jumps at you, it seizes you. But it isn't just violence. It can be very delicate, like Vermeer.' (Barche, 1990, p. 11).

And although he had already had a few exhibitions of his paintings in the US and in England, in Spring 1992 the municipality of Siena organized a large display of his works covering the period from 1938 to 1987. He very happily gave an interview on his own work to the art critic Gisela Barche (1990) who provided a thorough appreciation of his pictorial style

[14] He appears to think that the (now) so called Digital Divide will be a reality.

[15] 'As musicians compose in tone and rhythm, so I try to compose on form and colour' Goodwin wrote in the epigraph of Barche (1990).

and placed him in the contemporary art movement: ‘[his paintings] are, at the same time, precisely constructed in an exact observance of the colour wheel and with a repeated recourse to geometric forms, which could indifferently consist of circular or polyhedral forms or contours’ (ib., p. 12)

As Barche (ib, p. 13) wrote: ‘A comprehensive abstention from figurative objects [...] allows the viewer complete freedom to follow the shift of colour and form in a sort of optical interplay.’

The beauty of his art works parallels the elegance of his best economist’s models showing his talent in getting to the heart of the matter. Again in Barche’s words (ib., p. 14): ‘Possibly, it is his general approach to economics and painting that provides the unifying link: his endless curiosity to explore different possibilities and his will to always find the best possible solution to the problem at hand.’ This in spite of the fact that Goodwin denied any relationship between his being an economist and a painter. Also in his paintings he shows a complete freedom to express himself both in aesthetic solutions and in working methods and materials, not being constrained by the requirements of the art market.

The XV century old monastery that hosts the Faculty of Economics has many of his paintings hanging on the walls adding a touch of modern art to a medieval architecture and contributing to a more pleasant stay for students and faculty who work and study there: ‘The paintings are beautifully displayed in the high gothic hall with its soaring pillars which support the ribbed vaults. They offer the students a window open to another reality’ (ib., 14). And in Goodwin words: ‘The main thing I’m interested in is that my paintings should appeal in some way to other people as well. It’s like having children and then wanting them to go out into the world to make their own life. They are doing their work, I hope’ (ib.).

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